Problem Set 8
Finance 1, 47-720
(Bond pricing calibration)

1. Consider the following stochastic process for a pricing kernel (essentially, the Vasicek model):

\[- \log m_{t+1} = \frac{\lambda^2}{2} + z_t + \lambda \epsilon_{t+1},\]

where,

\[z_{t+1} = (1 - \varphi)\theta + \varphi z_t + \sigma \epsilon_{t+1},\]

and \(\epsilon_t \sim N(0, 1)\).

(a) Using the data provided on the course web page, which is not the same as the data in my paper, calibrate the model’s four parameters to the mean, unconditional variance, and autocorrelation of the short rate, and the level of the 10 year yield. Provide a graph of the mean yield curve for both the data and the theory.

(b) Provide a graph showing both the theoretical and the sample unconditional standard deviations of each yield-to-maturity.

(c) Provide a graph of what the theory predicts will be today’s yield curve. On the same graph, draw today’s actual yield curve. You will need some information source as http://www.bloomberg.com/markets/rates/index.html.

(d) Conduct a Monte Carlo simulation and provide a graph showing the different theoretical yield curve shapes which the model can generate.

2. Next, consider an alternative, based on Cox-Ingersoll and Ross:

\[- \log m_{t+1} = (1 + \frac{\lambda^2}{2})z_t + \lambda z_t^{1/2} \epsilon_{t+1},\]

where,

\[z_{t+1} = (1 - \varphi)\theta + \varphi z_t + \sigma z_t^{1/2} \epsilon_{t+1} .\]

(a) Work through the same calibration exercise as in question 1, and draw the same collection of graphs. Comment on any differences. If you find it constructive to consolidate some things on the same graph, that is fine.