Problem Set 7
Finance 1, 47-720

1. (dynamic representative agent construction)

Provide a formal proof of Proposition 2F in Duffie (bottom of page 26, second edition).

2. (dynamic spanning)

Consider the standard two-period binomial option pricing model.

- The initial stock price $S$ moves twice by a multiplicative factor, $u$ or $d$. Thus, there are three possible terminal values, $S_{uu}$, $S_{ud}$ and $S_{dd}$, and two possible intermediate values, $S_u$ and $S_d$.
- The probability of an uptick, $u$ is $p$.
- There is a riskless security which pays a (gross) of $R \equiv 1 + r$, per-period.
- There are European calls and puts, both with strike price $X$, which expire at the end of the second period.

(a) Demonstrate, in a rigorous manner, that markets are dynamically complete. Your answer should explicitly characterize the trading strategy involved in generating a contingent claim for period 2 consumption.

(b) Solve for the option values at each node on the event tree.

(b) Suppose that the options are American options. Characterize their prices.

(d) Use the following numbers. $S = 50$, $u = 1.5$, $d = .7$, $r = .02$, and $X = 55$. Solve for the put value at the initial node. Next, suppose that you are able to write someone 100 puts at a price of $18 per option. Characterize an arbitrage trading strategy and compute how profitable it is.