Currency Risk and Pricing Kernel Volatility

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Abstract

A basic tenet of lognormal asset pricing models is that a risky currency is associated with a *low* pricing kernel volatility. Empirical evidence implies that a risky currency is associated with a relatively *high* interest rate. Taken together, these two statements associate high-interest-rate currencies with low pricing kernel volatility. We document evidence suggesting that the opposite is true. We approximate the volatility of the pricing kernel with the volatility of the short-term interest rate. We find that, across currencies, *relatively* high interest rate volatility is associated with relatively high interest rates. This contradicts the prediction of lognormal models. One possible reason is that our approximation of the volatility of the pricing kernel is inadequate. We argue that this is unlikely, in particular for questions involving currencies. We conclude that lognormal models of the pricing kernel are inadequate for explaining currency risk and that future work should place increased emphasis on distributions that incorporate higher moments.

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1 Introduction

Currency risk is tricky. Unlike many financial securities, volatility and risk can be very different things. Consider the following anecdote. Producers of apples and bananas face supply shocks. The supply shocks become manifest in the relative price of fruit, the cost of one banana in units of apples. The producers, therefore, face price risk ... a positive supply shock to apples decreases the relative price of apples. Now consider volatility. Suppose that the apple shocks are volatile relative to the banana shocks. Apple producers notice two things: (i) the relative price of *bananas* isn't doing them any favors; it goes down whenever they get a bad shock, (ii) the supply shocks that they face are *dominating* the variability in the relative price. These apple producers therefore view the price of bananas as being *risky*. If you offer them a derivative security that has its value tied to the price of bananas, they will demand a risk premium in order to buy it. Banana producers, of course, face a reciprocal sort of situation. They notice that the relative price of *apples* goes against them when they get a bad shock. However, because their shocks are relatively small, point (ii) doesn't really apply to them. Therefore, if you offer them a derivative security tied to the price of *apples*, they will demand a *relatively* small risk premium in order to buy it. Herein lies the tricky business. The *high* volatility commodity, apples, enjoys relatively *low* price risk.

If you replace apples and bananas with the marginal utility of the domestic and the foreign representative agent, and the relative price of bananas with the price of one euro in units of U.S. dollars, you now understand what currency risk is in *any* lognormal model. The country with the highly-volatile marginal utility will have the currency with the low risk premium (in fact, negative). Algebraically,

$$E_t s_{t+1} - f_t = \frac{1}{2} (Var_t \log m_{t+1} - Var_t \log m_{t+1}^*) , \quad (1)$$

where s_t and f_t are the log spot and forward exchange rates (price of euros in units of dollars), m_t is the nominal marginal rate of substitution of the U.S. representative agent, and m_t^* is that of the European agent. The left-hand-side is the (continuously compounded) expected excess return on euros. We call it the *currency risk premium*. We call the nominal marginal rates of substitution, m_{t+1} and m_{t+1}^* , the domestic and foreign *pricing kernels*. Equation (1) must hold in any complete-markets equilibrium in which things are (conditionally) lognormally distributed.¹

¹Backus, Foresi, and Telmer (2001) page 287, Equation (14) (terse derivation below). More generally, Equation (1) must hold by virtue of no-arbitrage in the following sense. If one takes as given the process for m_{t+1} (the existence of which is guaranteed by no-arbitrage), and the rate of currency depreciation, then no-arbitrage guarantees the existence of a unique process for m_{t+1}^* such that, with conditional lognormality, Equation (1) must necessarily hold. Alternatively, take as given the *set* of all (no-arbitrage) pricing kernels, m and m^* , for, respectively, payoffs

Equation (1) says that if the dollar pricing kernel, m_{t+1} , has relatively *high* volatility, then the dollar will exhibit *low* risk in that its expected excess return over the euro will be negative. In more common language, the carry-trade funding (recipient) currencies are predicted to be those associated with high-(low)-volatility pricing kernels. This is the restriction that we seek to evaluate. Is currency risk the reciprocal of pricing kernel volatility?

Figure 1
Currency Risk and Return

The solid black line (left axis) reports the sample mean of the excess return on 5 interest-rate sorted currency portfolios, similar to that reported in Lustig, Roussanov, and Verdelhan (2011). The red dashed line (right axis) is, qualitatively, what lognormal pricing kernel models predict that pricing kernel volatility should look like.

Figure 1 illustrates our question graphically. It is an adaptation of Table 1 from Lustig, Roussanov, and Verdelhan (2011) (LRV). We construct five portfolios of currencies by constructing a monthly sort by the level of interest rates. Figure 1 reports the annualized average excess return, vis-a-vis the U.S. dollar, on each of the currency portfolios. We see what LRV, Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) and many others have documented; the ‘carry-trade’ of funding a long-position in a high-interest-rate currency by borrowing in a low-interest-rate

denominated in domestic and foreign currency units. Backus, Foresi, and Telmer (2001) (page 284, Proposition 1 and subsequent discussion) show that within the Duffie and Kan (1996) class of affine term structure models all members of this set are observationally equivalent given data on interest rates and exchange rates. This renders irrelevant the distinction between complete and incomplete markets. Finally, Brandt, Cochrane, and Santa-Clara (2006) show that Equation (1) must hold if we interpret m and m^* to be the unique least-squares projections of the set of no-arbitrage pricing kernels onto the space spanned by bond payoffs in multiple currencies. Their result also demonstrates the sense in which complete markets is not required for Equation (1) to be legitimate.

currency seems to pay a positive excess expected return. Our question, then, is this. If Figure 1 also plots, for each portfolio, the average pricing-kernel volatility of all the currencies in the portfolio, then, according to Equation (1), the line should be decreasing. Is it?

We find evidence suggesting that it is not. High expected return seems to be associated with high pricing kernel volatility. This implies one of two things. Either our method of computing the variances in Equation (1) is faulty, or there's something wrong with conditional lognormality. We argue that the latter is most likely. We conclude that models of currency risk should incorporate departures from lognormality. Our results are supportive of recent work by Brunnermeier, Nagel, and Pedersen (2008) and others that emphasize conditional skewness and leptokurtosis.

2 Notation and Basic Approach

The pricing kernel for claims denominated in U.S. dollars (USD) is m_{t+1} , so that $E_t m_{t+1} R_{t+1} = 1$ for all (gross) USD-denominated asset returns, R_{t+1} realized between dates t and $t + 1$. The analogous pricing kernel for claims denominated in foreign currency (say, British pounds, GBP) is m_{t+1}^* and GBP-denominated returns are R_{t+1}^* . Both m_t and m_t^* exist by virtue of no-arbitrage. One period, continuously-compounded USD and GBP interest rates are i_t and i_t^* so that

$$i_t = -\log E_t m_{t+1} \quad (2)$$

$$i_t^* = -\log E_t m_{t+1}^* . \quad (3)$$

The date- t nominal spot exchange rate, USD per GBP, is S_t . Since the USD pricing kernel must also price USD-denominated returns on GBP-denominated assets, no-arbitrage implies that

$$E_t \left(m_{t+1} \frac{S_{t+1}}{S_t} R_{t+1}^* \right) = 1 . \quad (4)$$

Since $E_t m_{t+1}^* R_{t+1}^* = 1$, then, if markets are complete,²

$$\frac{S_{t+1}}{S_t} = \frac{m_{t+1}^*}{m_{t+1}} . \quad (5)$$

This permits the (continuously-compounded) depreciation rate of USD to be written,

$$s_{t+1} - s_t = \log m_{t+1}^* - \log m_{t+1} , \quad (6)$$

²For what we'll do, the complete markets assumption is actually not required. See the footnote in the introduction.

where $s_t \equiv \log S_t$. The one-period forward exchange rate and its logarithm are F_t and f_t . Subtract Equation (3) from Equation (2) and invoke covered interest parity, $f_t - s_t = i_t - i_t^*$:

$$f_t - s_t = -\log E_t m_{t+1} - (-\log E_t m_{t+1}^*) . \quad (7)$$

Take the conditional mean of Equation (6), and subtract from it Equation (7):

$$\begin{aligned} E_t s_{t+1} - f_t &= (\log E_t m_{t+1} - E_t \log m_{t+1}) - (\log E_t m_{t+1}^* - E_t \log m_{t+1}^*) \\ &= \frac{1}{2} (\text{Var}_t \log m_{t+1} - \text{Var}_t \log m_{t+1}^*) , \end{aligned} \quad (8)$$

where the last equation holds by virtue of the conditional lognormality of m_t and m_t^* .³ It is the same as Equation (1) from the introduction. The LHS is the *risk premium on GBP*: the (continuously-compounded) expected excess return on borrowing USD and investing the proceeds in GBP. The RHS says that, in this class of models, a necessary condition for GBP to be risky is that its pricing kernel exhibit relatively *low* volatility.

Note that a great deal of empirical work — from older papers such as Domowitz and Hakkio (1985) to newer papers such as Chernov, Graveline, and Zviadadze (2012) — has focused on the conditional variance of the exchange rate, $\text{Var}_t(s_{t+1} - s_t)$. Similarly, Brunnermeier, Nagel, and Pedersen (2008) focus on $\text{Skew}_t(s_{t+1} - s_t)$. While these moments are certainly interesting in-and-of-themselves, the combination of Equations (6) and (8) indicate that they emphasize moments of the difference, whereas currency risk is more directly about the difference of the moments. Sorting this out (in a new way) is the focal point of our paper.

2.1 A Very Rough Approximation

Equation (8) tells us that the currency risk premium is the difference between two unobservable variables. Many papers, of course, have written down an explicit (lognormal) model of m_t and m_t^* under which this difference is a function of observables. A successful model has been one with an interest rate differential, $i_t - i_t^*$, that is *negatively* correlated with the volatility difference, $\text{Var}_t \log m_{t+1} - \text{Var}_t \log m_{t+1}^*$. Our goal, instead, is to derive an empirical measure of this variance difference that applies more generally than to just one, particular (lognormal) model. We begin with a very informal derivation, intended to get at the main idea. In the next section we provide a tight, formalized analysis showing that what we do here applies much more generally.

³Backus, Foresi, and Telmer (2001), page 286, Equation (12), shows the more general expression, involving higher-order moments. This will play a key role later, in Section #.

With lognormality, Equation (2) implies that the period-ahead domestic interest rate is

$$i_{t+1} = -E_{t+1} \log m_{t+2} - \frac{1}{2} \text{Var}_{t+1} \log m_{t+2} . \quad (9)$$

Suppose that variation in conditional mean is relatively small. This is not as unreasonable as it might sound, at least for our question. Backus, Foresi, and Telmer (2001) show that Fama’s (1984) necessary conditions for resolving the forward premium anomaly translate into conditions stating that ‘variance in pricing kernel variance must be larger than variance in pricing kernel means.’ Papers by ? and ? show that something similar is implied by the Campbell-Shiller, Fama-Bliss term structure regressions. Thus ignoring the first term in Equation (9) we have

$$\text{Var}_t i_{t+1} \approx \frac{1}{4} \text{Var}_t \text{Var}_{t+1} \log m_{t+2} .$$

Suppose that the conditional variance of the conditional variance isn’t too different from the conditional variance. For some processes, this is sensible. For example, for the canonical square-root process, $x_{t+1} = \mu + \varphi x_t + \sigma x_t^{1/2} \varepsilon_{t+1}$, we have that $\text{Var}_t \text{Var}_{t+1} x_{t+2} = \sigma^2 \text{Var}_t x_{t+1} = \sigma^4 x_t$. For other processes — *e.g.*, an autoregression with stochastic volatility, but where the volatility is homoskedastic — it makes no sense. Ignoring the latter,

$$\text{Var}_t i_{t+1} \approx \frac{1}{4} \text{Var}_t \log m_{t+1} . \quad (10)$$

This is the main idea. Perhaps we can learn something about pricing kernel volatility by simply estimating interest rate volatility?

Suppose that this is valid. Substitute it into Equation (8):

$$E_t s_{t+1} - f_t \approx 2(\text{Var}_t i_{t+1} - \text{Var}_t i_{t+1}^*) .$$

In most lognormal models the LHS is a linear function of the interest rate differential:

$$a + b(i_t - i_t^*) \approx 2(\text{Var}_t i_{t+1} - \text{Var}_t i_{t+1}^*) . \quad (11)$$

There is overwhelming empirical evidence indicating that $b < 0$. This is the ‘Uncovered Interest Rate Parity (UIP)’ regression evidence first found by Bilson (1981), Fama (1984) and Tryon (1979). It is the basis for the foreign currency ‘carry trade.’ Assume that $a = 0$.⁴ Equation (11) implies that, according to lognormal models, (i) the interest rate differential, $i_t - i_t^*$ and the volatility differential,

⁴This is implied by the UIP evidence if (i) $b = -1$, (ii) UIP holds unconditionally, and (iii) the unconditional mean of the interest rate differential is zero, $E(i_t - i_t^*) = 0$. For many currency pairs all of these conditions are empirically plausible (Engel (2011) provides some up-to-date evidence and his survey paper, Engel (1996), is a standard reference for a more exhaustive survey). For those for which they are not, our story is basically unchanged once we subtract out an innocuous mean.

$Var_t i_{t+1} - Var_t i_{t+1}^*$, should have the opposite sign, and (ii) they should be negatively correlated.

Figure 3 shows results that are representative of our main findings. It shows that, for a classic ‘carry trade’ pair of currencies — USD and the Australian dollar (AUD) — restrictions (i) and (ii) seem to be strongly at odds with the data. Much more often than not, the LHS and RHS of Equation (11) have the *same* sign. Moreover, they are positively correlated at 0.52.

Interest rate differentials and interest rate volatility differentials appear to be positively related. The high-interest rate currency — the carry-trade recipient currency that pays a positive risk premium — also appears to be the high interest-rate-volatility currency. This isn’t the case for every currency pair and time period but, as we exhaustively demonstrate in Section 5, it is much more the rule than the exception. Our approximation, Equation (10), suggests that we can restate this. The high-interest rate currency appears to be the high pricing-kernel-volatility currency. This is a stark contradiction of any lognormal model of currency risk.

We now tighten up the approximation from Section 2.1 and show precisely under what conditions high interest rate volatility is necessarily associated with high pricing kernel volatility.

3 Affine Models

The Duffie and Kan (1996) class of lognormal, affine pricing kernel models can be specified as follows. Uncertainty in the domestic country is described by the k -dimensional vector of state variables z that follows a square-root model:

$$z_{t+1} = (I - \Phi)\theta + \Phi z_t + \Sigma(z_t)^{1/2}\epsilon_{t+1} \quad ,$$

where $\{\epsilon\} \sim \text{NID}(0,1)$, $\Sigma(z_t)$ is a $k \times k$ diagonal matrix with a typical element given by $\sigma_i(z_t) = \alpha_i + \beta_i^\top z_t$, where β_i has nonnegative elements, and Φ is a $k \times k$ stable matrix with positive diagonal elements. The process for z requires that the volatility functions, $\sigma_i(z)$, be positive, which places additional restrictions on the parameters. The pricing kernel is

$$-\log m_{t+1} = \delta + \gamma^\top z_t + \lambda^\top \Sigma(z_t)^{1/2} \epsilon_{t+1} \quad , \quad (12)$$

where the $k \times 1$ vector γ is referred to as the “factor loadings” for the pricing kernel, and the $k \times 1$ vector λ is referred to as the “price of risk” vector.

Using Equation (2), together with the dynamics of the pricing kernel from Equation (12), the interest rate is

$$i_{t+1} = \left(\delta - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \alpha_j \right) + \left(\gamma^\top - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_j^\top \right) z_{t+1} \quad .$$

The conditional variance of the home pricing kernel is

$$Var_t(m_{t+1}) = \left(\lambda^\top \Sigma(z_t)^{1/2} \right) Var_t(\epsilon_{t+1}) \left(\lambda^\top \Sigma(z_t)^{1/2} \right)^\top = \lambda^\top \Sigma(z_t) \lambda = \sum_{j=1}^k \lambda_j^2 \sigma_j(z_t) \quad ,$$

and the conditional variance of the home interest rate is

$$\begin{aligned} Var_t(i_{t+1}) &= \left(\gamma^\top - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_j^\top \right) \Sigma(z_t) \left(\gamma^\top - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_j^\top \right)^\top \\ &= \sum_{n=1}^k \left(\gamma_n - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_{n,j} \right)^2 \sigma_n(z_t) \quad . \end{aligned}$$

Similarly to the domestic country, the foreign country is described by a k -dimensional vector z^* . Foreign parameters are denoted with an asterisk. It is straightforward to derive the expressions for the foreign pricing kernel, m^* and the foreign interest rate, i^* .

3.1 Theorems

Under what conditions can we say that

$$Var_t(m_{t+1}) > Var_t(m_{t+1}^*) \Leftrightarrow Var_t(i_{t+1}) > Var_t(i_{t+1}^*) \quad ? \quad (13)$$

The validity — or not — of result (13) is in general parameter dependent. However, for two special cases which are ubiquitous in the term structure and the currency risk premium literature, it turns out that we can say a fair bit.

Theorem 1: *Symmetric Coefficients*

Let Ω (Ω^*) denote the vector containing the home (foreign) parameters. Let $\Omega = \Omega^*$, $\lambda \neq 0$ and $\left(\gamma^\top - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_j^\top \right) \neq 0$. Then $(\Sigma(z_t) - \Sigma(z_t^*))$ is positive definite if and only if $Var_t(m_{t+1}) > Var_t(m_{t+1}^*)$, if and only if $Var_t(i_{t+1}) > Var_t(i_{t+1}^*)$.

The proof is in Appendix A. Theorem 1 says that, with symmetric coefficients, a relatively large conditional variance of the home state variables is associated with a relatively large conditional variance of the home pricing kernel and a relatively large conditional variance of the home interest rate. A given ranking in the conditional variance of the state variables in each country is associated with the same ranking

in the conditional variance of the pricing kernels and the conditional variance of the interest rates.

As an example, consider the single-factor (per-country) case, $k = 1$. Theorem 1 simplifies to

$$\text{Var}_t(z_{t+1}) > \text{Var}_t(z_{t+1}^*) \Leftrightarrow \text{Var}_t(m_{t+1}) > \text{Var}_t(m_{t+1}^*) \Leftrightarrow \text{Var}_t(i_{t+1}) > \text{Var}_t(i_{t+1}^*) ,$$

which can be read as “high conditional variance at home means high conditional variance in the home kernel and high conditional variance in the home interest rate.”

Theorem 2: *Common Factors*

Consider the case of common factors, $z_t = z_t^*$. Assume that $\gamma = \gamma^*$, $\beta = \beta^*$, and that there exists a strong enough ‘precautionary savings motive’ associated with both the domestic and foreign pricing kernels, so that

$$\gamma^\top - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_j^\top < 0 \quad \text{and} \quad \gamma^\top - \frac{1}{2} \sum_{j=1}^k (\lambda_j^*)^2 \beta_j^\top < 0 .$$

Suppose that the prices of risk λ and λ^* can be *ordered*, so that either $\lambda > \lambda^*$ or $\lambda < \lambda^*$. Then, $|\lambda| > |\lambda^*|$ if and only if $\text{Var}_t(m_{t+1}) > \text{Var}_t(m_{t+1}^*)$ if and only if $\text{Var}_t(i_{t+1}) > \text{Var}_t(i_{t+1}^*)$.

The proof is in Appendix A. Theorem 2 says that, with common factors, a sufficient condition for the conditional variance of the domestic pricing kernel to be larger than the conditional variance of the foreign pricing kernel is that the price of risk of each of the home state variables is at least as large (in absolute value) than the price of risk of each of the foreign state variables. When this is the case, a strong enough precautionary saving demand in each country delivers a larger conditional variance of the home interest rate, relative to the conditional variance of the foreign interest rate. For the theorem to hold, symmetry in the factor loadings and in the sensitivity of the conditional variances to the state variables is required.

The condition that $\gamma^\top - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_j^\top < 0$ (and its foreign counterpart) is usually referred to in the literature as a *strong enough precautionary savings motive*. Lustig, Roussanov, and Verdelhan (2011) is an example of Theorem 2 at work. They show that an affine model with a common factor, common coefficients across countries with the exception of the price of risk, and a precautionary saving motive that is strong enough in each country delivers interest rate and exchange rate dynamics that are consistent with the carry trade facts. Hence, the assumptions

of Theorem 2 are not stringent at all. They *must* be satisfied for an affine model of the Duffie-Kan class to fit the data.

As an example, consider the single factor case, $k = 1$. Here, a stronger result is available. Under the conditions of Theorem 2

$$|\lambda| > |\lambda^*| \Leftrightarrow \text{Var}_t(m_{t+1}) > \text{Var}_t(m_{t+1}^*) \Leftrightarrow \text{Var}_t(i_{t+1}) > \text{Var}_t(i_{t+1}^*) \quad ,$$

where now λ and λ^* are scalars. In words, in a world with one single common factor, when all other coefficients are symmetric, having a relatively large home price of risk (in absolute value) means having a relatively high domestic kernel volatility and a relatively high interest rate volatility.

4 Data and Estimation

For our main analysis, we use data for 114 countries on 3-month treasury bill yields, forward and spot exchange rates from *Global Financial Data* (GFD) for the period 1950-2009. As a comparison, we also use eurocurrency interest rates for 27 countries from the *Financial Times/ICAP* (FT/ICAP), 1975-2009. Appendix B provides details. The number of countries is extensive, but, obviously, we do not have data for all countries and time periods, and the relevance of many of the countries for our question is questionable. We therefore report results for a number of different subsamples of time and country. Our main result is not sensitive to either.

We estimate a GARCH model for interest rate volatility. Details on the estimation procedure are given in Appendix B. We report results for both bilateral currency pairs and portfolios of currencies. The latter are formed in a manner identical to Lustig, Roussanov, and Verdelhan (2011), by sorting on interest rate levels at a monthly frequency.

5 Results

In the introduction we used Figure 1 to illustrate our main question: is pricing kernel volatility decreasing in the interest rate differential, as lognormal theory predicts it should? Figure 2 answers our question. We conduct the exact same exercise as Lustig, Roussanov, and Verdelhan (2011). We sort currencies by interest rate and, each month, form 5 portfolios. Figure 2 plots the time-averaged excess return on each portfolio, and the time-averaged interest rate volatility. Both are increasing as we move from low to high interest rate portfolios. The answer to the question is no.

Figure 2
High Interest Rates are Associated with High Interest Rate Volatility

The solid black line (left axis) is the same as that in Figure 1 from the introduction. It is the sample mean of the excess return on 5 interest-rate sorted currency portfolios, similar to that reported in Lustig, Roussanov, and Verdelhan (2011). The red dashed line (right axis) is, the time-averaged interest rate volatility on each of the portfolios. The class of lognormal models described in Section 3 restrict the red line to be decreasing if the black line is increasing. The data are, therefore, inconsistent with lognormal models in this dimension.

The remainder of our empirical work simply consists of further substantiating the point of Figure 2 by considering different samples and data sources. Figure 4 shows average volatility for the currency portfolios for a number of different subsamples, starting from 1950, 1975, 1987 and 1995. Figure 5 reports analogous results for bilateral currency pairs. Figure 6 compares results across the GFD and FT/ICAP data sources. In all cases, we see robust evidence of high interest rates differentials being associated with high interest rate volatility differentials.

6 Conclusions

We have shown that Gaussian models of currency risk face an empirical challenge. A high interest rate differential seems to be associated with a high interest-rate *volatility* differential. In a broad class of Gaussian models of the pricing kernel, this is inconsistent with (i) the empirical observation that high interest rates are associated with high excess expected returns, and (ii) the theoretical restriction that high excess expected returns accrue to the *low* volatility pricing kernel. This inconsistency derives from some conditions under which interest rate volatility and pricing kernel volatility are positively related. These conditions are not exhaustive. But they are satisfied by some models that are prominent in the literature. This suggests that we either enrich these models, or consider non-Gaussian alternatives.

We’ve couched our analysis in terms of trying to observe *directly* the difference in the moments of pricing kernels. This stands in contrast to much existing work, which tends to be based on moments of the difference in pricing kernels (*e.g.*, the variance of the exchange rate). We find our approach useful in that it emphasizes something fundamental about what currency risk *is*, while at the same time providing some links to other asset pricing models and results such as those from the literature on the term structure of interest rates. Nevertheless, we admit that ours does have the flavor of many previous papers that have gone searching for the unicorn by trying to identify the magical pricing kernel using very few assumptions and very little data (*i.e.*, asset return data only). An alternative interpretation, therefore, is as follows.

Ever since Bilson (1981), Fama (1984) and Tryon (1979) discovered the existence of excess expected returns in currency markets, many models have been developed to account for this behavior. The first models focused almost exclusively on the UIP regression coefficient. For subsequent models the bar has been raised higher. Other moments of the joint distribution of exchange rates, interest rates, consumption and so on have been emphasized. Sharpe ratios on traded currency *portfolios* have been emphasized. Our results can be viewed as simply suggesting one more moment: the interest-rate volatility differential. There is clear evidence on this, and it places very binding restrictions on state-of-the-art models. Moreover, it seems (to us) to be a particularly important moment. There is a clear link to theory and this link might be pointing us to non-Gaussian behavior, something that has been strongly emphasized in the recent literature on crashes, disasters and the like. We like this moment.

Finally, we close with a broad, interpretive point. Currency risk is a particular type of “change of units risk.” In Gaussian models, an inescapable characteristic of this is that high risk is associated with low volatility. If the units that I care about are subject to *relatively* volatile shocks, then financial securities with payoffs denominated in these units will pay a *negative* risk premium (relative to the other

units in question). Our goal has been to ask if this characteristic fits the facts. To find out, one must first take a stand on what are “the units that I care about.” The obvious answer (to an economist) is “real marginal utility units,” where the word “real” makes clear that this is “marginal utility per unit of *goods*.” We have circumvented this, focusing instead on *nominal* marginal utility units: “utils per dollar.” We’ve done this because most financial securities are denominated in dollars, not in goods. We must therefore be silent on whether or not Gaussian models of *real* exchange rates fit the facts. But we can say something about nominal models, which, one-way-or-another, have been most prevalent in the literature. We find evidence suggesting that nominal exchange rate risk is more than just a Gaussian phenomenon. We also note that, while our data have not addressed them, Gaussian models of real marginal utility have become more and more prevalent in the literature ever since Hansen, Heaton, and Li (2008) developed their linearization of the recursive class of preferences. Much of the long-run risk literature initiated by Bansal and Yaron (2004) features models that are conditionally Gaussian. A prominent example featuring exchange rates is Bansal and Shaliastovich (2013). Our approach is easily applied in this setting, and will feature relative consumption volatility in addition to interest rate volatility. This is work-in-progress.

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A Appendix: Proofs of Theorems

Proof of Theorem 1

Note that, since $(\Sigma(z_t) - \Sigma(z_t^*))$ is diagonal, positive definiteness requires $\sigma_j(z_t) \geq \sigma_j(z_t^*)$, for every j , with at least one strict inequality (i.e. $\exists i$ such that $\sigma_i(z_t) > \sigma_i(z_t^*)$). Now, when $\Omega = \Omega^*$, we have

$$\text{Var}_t(m_{t+1}) - \text{Var}_t(m_{t+1}^*) = \lambda^\top (\Sigma(z_t) - \Sigma(z_t^*)) \lambda \quad .$$

Therefore, by the definition of positive definiteness, we have

$$(\Sigma(z_t) - \Sigma(z_t^*)) \text{ is positive definite } \Leftrightarrow \text{Var}_t(m_{t+1}) > \text{Var}_t(m_{t+1}^*) \quad ,$$

whenever $\lambda \neq 0$. Similarly,

$$\text{Var}_t(i_{t+1}) - \text{Var}_t(i_{t+1}^*) = \left(\gamma^\top - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_j^\top \right) (\Sigma(z_t) - \Sigma(z_t^*)) \left(\gamma^\top - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_j^\top \right)^\top \quad .$$

Again, positive definiteness gives

$$(\Sigma(z_t) - \Sigma(z_t^*)) \text{ is positive definite } \Leftrightarrow \text{Var}_t(i_{t+1}) > \text{Var}_t(i_{t+1}^*) \quad ,$$

whenever $\left(\gamma^\top - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_j^\top \right) \neq 0$. Last, let $a \neq 0$ be any $k \times 1$ vector and note that the matrix $(\Sigma(z_t) - \Sigma(z_t^*))$ is diagonal. Therefore,

$$a^\top (\Sigma(z_t) - \Sigma(z_t^*)) a = \sum_{j=1}^k a_j^2 (\sigma_j(z_t) - \sigma_j(z_t^*)) > 0 \Leftrightarrow \sigma_j(z_t) \geq \sigma_j(z_t^*), \text{ for every } j \quad ,$$

with at least one strict inequality associated with a non zero element of a . $\square$

Proof of Theorem 2

$$\text{Var}_t(m_{t+1}) - \text{Var}_t(m_{t+1}^*) = (\lambda^2 - (\lambda^*)^2)^\top \text{diag}\{\Sigma(z_t)\} \quad ,$$

which, under the assumption that λ and λ^* can be ordered, is positive if and only if $\lambda^2 > (\lambda^*)^2$, that is $|\lambda| > |\lambda^*|$. Moreover,

$$\text{Var}_t(i_{t+1}) - \text{Var}_t(i_{t+1}^*) = \left(\left(\gamma^\top - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_j^\top \right)^2 - \left(\gamma^\top - \frac{1}{2} \sum_{j=1}^k (\lambda_j^*)^2 \beta_j^\top \right)^2 \right)^\top \text{diag}\{\Sigma(z_t)\} \quad ,$$

which, under the conditions that $\gamma = \gamma^*$, $\beta = \beta^*$, $\gamma^\top - \frac{1}{2} \sum_{j=1}^k \lambda_j^2 \beta_j^\top < 0$, and $\gamma^\top - \frac{1}{2} \sum_{j=1}^k (\lambda_j^*)^2 \beta_j^\top < 0$ is positive if only if $\lambda^2 > (\lambda^*)^2$, that is $|\lambda| > |\lambda^*|$. $\square$

B Appendix: Data

B.1 Interest Rates

Our main analysis is done with 3-month treasury bill yields for 114 countries from *Global Financial Data* (GFD) for the period 1950-2009 as it is the most comprehensive interest rate data available. As a robustness check, we also use eurocurrency interest rates from FT/ICAP through Datastream. FT/ICAP data consists of data for 27 mostly developed countries where for many of them data starts in 1975.

Here we document the availability of various interest rates data from Datastream and why we thought FT/ICAP was the best interbank interest rate data available. In Datastream, the two sources for eurocurrency interest rates are FT/ICAP and ICAP while the British Bankers Association (BBA) gives the London interbank rates (LIBOR).⁵ Each have different set of countries and data years.

In 2006, Financial Times (FT) stopped providing euro currency rates to Datastream for the FT/ICAP series (for which Datastream mnemonic all starts with “EC”) series. So Datastream continued these same series by splicing them with Intercapital (or ICAP, but formerly Garban Information Services (GS)) data so that starting in 2006, FT/ICAP and ICAP series are identical. For the countries for which GS is not available (e.g. Hong Kong, Singapore, South Africa), Tullett Prebon (TP) data was used instead. The TP data itself starts in 2006. The source column in Datastream’s *Navigator* is not very informative. If the name of the series has FT/ICAP/TR, the source column will say “Thomson Reuters” but the source is really FT spliced with ICAP. If the name series has FT/TP, the source column appropriately says Tullett Prebon. The ones that have “dead” in the name have FT as the source and are dead series because neither ICAP nor TP has data on these countries. In Table 1, the countries of these dead series start with Greece and end with Thailand. For all the ICAP series (which start with “GS....”), the source column says “Thomson Reuters” but it is actually all ICAP.

A Datastream documentation file regarding this discontinuation of FT series gives alternative series that can be used instead. This list includes above mentioned ICAP, Tullett Prebon, BBA LIBORs, as well as “locally supplied” rates which are interbank rates from mainly national sources. These rates from national sources can be found under “National Interest Rates.” The column “Alt IB” in Table 1 demonstrates the starting dates of the specific series that were listed in this document although there can be more than one interbank type rate under “National Interest Rates”.

⁵There is also Tullett Prebon through Datastream that provides eurocurrency rates, but their data starts in 2006 only.

Datastream has various interest rate data under "National Sources." In the future, our treasury bill yields data from GFD could be potential supplemented with the combination of Eurocurrency rates (mainly, FT/ICAP) and interbank rates from national sources. A Datastream documentation about risk free rates discusses this:

A risk free interest rate is the internal rate of return that can be obtained by investing in a financial instrument without (or very limited) credit risk. Normally this will relate to a short term investment in a financial instrument backed by the government. These money market securities bear no credit risk and have a limited re-investment risk, when the investment is rolled over for another short term period. In general we recommend the (annualised) yield on 3-month treasury bills, as the best instrument to use for any analysis involving risk free rates. However, for currencies where no liquid treasury bill market exists (or this market is subject to institutional distortions), interbank rates such as LIBOR or EURIBOR rates can be used. These do, however, bear a minimal credit risk inherent to the banks active in the market. Currencies with liquid repo-markets where the general collateral is a risk free long-term government bond, offer another alternative for an interest rate which comes closest to 'risk-free', but are not available for as many currencies as interbank rates. A good example of this latter alternative is the Japanese 'Gensaki' market. The series we recommend for the main markets are detailed on the following table.

Column titled "Risk Free" in Table 1 demonstrates starting dates for the series specified in the document. Not shown in the table are Datastream's recommendations for risk free rates on Russia, China, Korea, Pakistan, Taiwan, Argentina, Brazil, Chile, Mexico and Venezuela also. Overall, this recommended list is not comprehensive (even few of the series above is suspended) and one would have to manually go through the various interest rates available for each country under "National Interest Rates" and pick the most relevant rate (most likely some interbank rate is the best) in order to supplement GFD data.

Lastly, for the seven (for BBA, it is six) euro legacy countries, since the introduction of the Euro in 1999, GS, EC, and BBA interest rates are identical across these countries. Obviously, we will not have the same problem with GFD treasury yields data.

Market	FT/ICAP	ICAP	BBA	Alt IB	"Risk Free"	GFD		
	Start	Start	Start	Start	Start	Start	End	nmiss
Australia	1997	1988	1986	1987	1977	1950	2009	3
Canada	1975	1995	1990	1992	1981	1950	2009	1
Denmark	1985	1995	2003	1989	1993	1976	2007	29
Hong Kong	1997			1986		1991	2009	1
Japan	1978	1995	1986	1996		1960	2009	1
New Zealand	1997	1988	2003	1987	1989	1978	2009	3
Norway	1997	1995		1986	1986	1984	2009	1
Singapore	1988			1987	1989	1987	2009	1
South Africa	1997			2000		1950	2009	1
Sweden	1997	1995	2006	1993	1994	1955	2009	1
Switzerland	1975	1995	1986	1974	1950	1980	2009	3
United Kingdom	1975	1995	1986	1975	1972	1950	2009	1
United States	1975	1995	1986	1971	1955	1950	2009	1
Other Western European	1999	1999	1998	1999	1999	1984	2009	2
Belgium	1978	1995				1950	2009	2
France	1975	1995	1989			1950	2009	61
Germany	1975	1995	1986			1953	2009	2
Italy	1978	1995	1990			1950	2009	2
Netherlands	1975	1995	1991			1950	2009	2
Portugal	1992	1995	1994			1985	2009	2
Spain	1992	1996	1990			1982	2009	2
Greece	1999					1980	2009	2
India	1999			1999	1994	1993	2009	1
Indonesia	1997			2001	1985	2000	2003	72
Malaysia	1997			1994	1998	1961	2009	1
Philippines	1999					1976	2009	3
Thailand	1997			2005	1995	1977	2009	88
Total	27	18	16	18	15	27		

Table 1: This table compares various interest rates data available from Datastream (the first five columns) with GFD data. "nmiss" stands for the number of missing months

Market	Start	End	nmiss	Market	Start	End	nmiss
ALBANIA	1994	2009	4	KOREA, REPUBLIC OF	1987	2009	2
ALGERIA	1998	2009	5	KUWAIT	1979	2005	59
ANGOLA	2000	2009	3	KYRGYZSTAN	1994	2009	3
ARGENTINA	2002	2009	16	LATVIA	1994	2008	16
ARMENIA	1995	2009	3	LEBANON	1977	2009	4
AUSTRIA	1960	1990	228	LITHUANIA	1994	2009	4
AZERBAIJAN	1997	2009	9	MACEDONIA	1997	2009	3
BAHAMAS	1971	2009	3	MADAGASCAR	2000	2009	3
BAHRAIN	1987	2009	3	MALTA	1987	2009	3
BANGLADESH	1984	2009	29	MAURITIUS	1996	2009	3
BARBADOS	1966	2009	5	MEXICO	1978	2009	6
BELIZE	1978	2009	4	MOLDOVA, REPUBLIC OF	1995	2009	3
BOLIVIA	1994	2009	6	MONGOLIA	2006	2008	20
BOTSWANA	1996	2009	3	MONTENEGRO	2004	2009	3
BRAZIL	1965	2009	62	MOROCCO	2008	2009	4
BULGARIA	1992	2008	23	MOZAMBIQUE	2000	2009	4
BURUNDI	2001	2009	29	NAMIBIA	1991	2009	8
CAPE VERDE	1998	2009	3	NEPAL	1981	2008	17
CENTRAL AFRICAN REPUBLIC	1996	2004	139	NICARAGUA	2003	2007	24
CHILE	1997	2009	2	NIGERIA	1970	2009	229
CHINA	2002	2009	1	PAKISTAN	1991	2009	1
COLOMBIA	1998	2009	2	POLAND	1991	2009	1
COSTA RICA	1996	2009	3	ROMANIA	1994	2005	51
CROATIA	2000	2009	3	RUSSIAN FEDERATION	1994	2009	18
CYPRUS	1975	2008	21	RWANDA	1999	2009	5
CZECH REPUBLIC	1993	2009	5	SAUDI ARABIA	1991	2008	12
EGYPT	1991	2009	1	SERBIA	1997	2009	34
EL SALVADOR	2001	2005	51	SIERRA LEONE	1965	2008	12
ETHIOPIA	1985	2008	12	SLOVAK REPUBLIC	1993	2007	24
FIJI	1975	2009	4	SLOVENIA	1998	2009	5
GEORGIA	2001	2005	54	SRI LANKA	1981	2009	1
GHANA	1978	2009	9	SWAZILAND	1981	2006	45
GUYANA	1972	2009	6	TAIWAN	1974	2009	3
HAITI	1996	2009	2	TANZANIA, UNITED REPUBLIC OF	1993	2009	5
HONDURAS	1998	2000	108	TRINIDAD AND TOBAGO	1964	2009	7
HUNGARY	1988	2009	3	TUNISIA	1990	2009	2
ICELAND	1987	2009	2	TURKEY	1985	2009	3
IRAQ	2004	2009	3	UGANDA	1980	2009	3
IRELAND	1969	2009	2	URUGUAY	1992	2009	49
ISRAEL	1992	2009	2	VENEZUELA	1996	2003	72
JAMAICA	1953	2009	3	VIET NAM	1997	2009	9
JORDAN	2000	2009	36	ZAMBIA	1978	2009	3
KAZAKHSTAN	1994	2009	2	ZIMBABWE	1962	2009	11
KENYA	1972	2009	8				

Table 2: The remaining 87 GFD interest rates (mainly treasury yields). “nmiss” stands for number of missing months.

B.2 Exchange Rate Data

Our main exchange rate data is from GFD for 172 countries for the period 1950-2009. This dataset is most comprehensive compared to three different data sources of forward and spot exchange rates available through Datastream: Barclays Bank PLC (BBI), Tenfore, and WM/Reuters (WMR). For each of these sources, full sample of countries and data years vary and are a lot more limited compared to GFD as shown in the table belows.

	WM/R	Tenfore	Barclays		WM/R	Tenfore	Barclays
Argentine Peso	2004			Latvian Lat	2004		
Australian Dollar	1996	1990	1984	Lithuanian Lita	2004		
Austrian Schilling	1996			Malaysian Ringgit	1996		
Belgian Franc	1996			Maltese Lira	2004		
Brazilian Real	2004			Mexican Peso	1996		
Bulgarian Lev	2004			Moroccan Dirham	2004		
Canadian Dollar	1996	1990	1984	New Zealand Dollar	1996	1990	1984
Chilean Peso	2004			Norwegian Krone	1996	1990	1984
Chinese Yuan Renminbi	2002			Omani Rial	2004		
Colombian Peso	2004			Pakistani Rupee	2004		
Croatian Kuna	2004			Peruvian Nuevo Sol	2004		
Cyprian Pound	2004			Philippine Peso	1996	2006	
Czech Koruna	1996	1996		Polish Zloty	2002	1996	
Danish Krone	1996	1990	1984	Portuguese Escudo	1996		
Dutch Guilder	1996			Qatari Riyal	2004		
Egyptian Pound	2004			Romanian Leu	2004	2008	
Estonian Kroon	2004			Russian Federation Rouble	2004		
Euro	1998	1990	1999	Saudi Arabian Riyal	1996	1990	
Finnish Markka	1996			Singaporean Dollar	1996	1990	1984
French Franc	1996			Slovak Koruna	2002		
German Mark	1996			Slovenian Tolar	2004		
Greek Drachma	1996			South African Rand	1996	1990	1983
Hong Kong Dollar	1996	1990	1983	Spanish Peseta	1996		
Hungarian Forint	1997			Swedish Krona	1996	1990	1984
Icelandic Krona	2004	2006		Swiss Franc	1996	1990	1983
Indian Rupee	1997			Taiwanese Dollar	1996		
Indonesian Rupiah	1996			Thai Baht	1996	1995	
Irish Punt or Pound	1996			Tunisian Dinar	2004		
Israeli Sheqel	2004	2006		Turkish Lira	1996	2006	
Italian Lira	1996			Ukrainian Hryvnia	2004	2008	
Japanese Yen	1996	1990	1983	United Arab Emirates Dirham	1996	1995	
Jordanian Dinar	2004			United Kingdom Pound	1996	1990	1983
Kazakh Tenge	2004			Venezuelan Bolivar	2004		
Kenyan Shilling	2004						
Korean Won	2002			Total	69	25	13
Kuwaiti Dinar	1996	1990					

Table 3: Exchange rate data sources from Datastream

B.3 GARCH Model

Interest rates, r_t , are AR(p) process:

$$r_t = c + \beta_1 r_{t-1} + \dots \beta_p r_{t-p} + u_t$$

where u_t is $GARCH(r, m)$:

$$u_t \sim N(0, \sigma_t^2) \quad (14)$$

$$\sigma_t^2 = \kappa + \sum_{i=1}^r \delta_i \sigma_{t-i}^2 + \sum_{i=1}^m \alpha_i u_{t-i}^2 \quad (15)$$

For all countries, we first fit AR(5) to the interest rates, and fit the residuals to GARCH(1,1).⁶

⁶In the future, we will update our procedure to estimate the parameters of AR and GARCH processes at the same time.

	all	1975	1987	1995		all	1975	1987	1995
ALB	1	0	0	1	KOR	1	0	1	1
DZA	1	0	0	0	KWT	1	0	1	1
AGO	1	0	0	0	KGZ	1	0	0	1
ARG	1	0	0	0	LVA	1	0	0	1
ARM	1	0	0	0	LBN	1	0	1	1
AUS	1	1	1	1	LTU	1	0	0	1
AUT	1	1	1	0	MKD	1	0	0	0
AZE	1	0	0	0	MDG	1	0	0	0
BHS	1	1	1	1	MYS	1	1	1	1
BHR	1	0	0	1	MLT	1	0	0	1
BGD	1	0	1	1	MUS	1	0	0	0
BRB	1	1	1	1	MEX	1	0	1	1
BEL	1	1	1	1	MDA	1	0	0	0
BLZ	1	0	1	1	MNG	1	0	0	0
BOL	1	0	0	1	MNE	1	0	0	0
BWA	1	0	0	0	MAR	1	0	0	0
BRA	1	1	1	1	MOZ	1	0	0	0
BGR	1	0	0	1	NAM	1	0	0	1
BDI	1	0	0	0	NPL	1	0	1	1
CAN	1	1	1	1	NLD	1	1	1	1
CPV	1	0	0	0	NZL	1	0	1	1
CAF	1	0	0	0	NIC	1	0	0	0
CHL	1	0	0	0	NGA	1	0	0	1
CHN	1	0	0	0	NOR	1	0	1	1
COL	1	0	0	0	PAK	1	0	0	1
CRI	1	0	0	0	PHL	1	0	1	1
HRV	1	0	0	0	POL	1	0	0	1
CYP	1	1	1	1	PRT	1	0	1	1
CZE	1	0	0	1	ROU	1	0	0	1
DNK	1	0	1	1	RUS	1	0	0	1
EGY	1	0	0	1	RWA	1	0	0	0
SLV	1	0	0	0	SAU	1	0	0	1
ETH	1	0	1	1	SRB	1	0	0	0
EUR	1	0	1	1	SLE	1	1	1	1
FJI	1	1	1	1	SGP	1	0	0	1
FRA	1	1	1	1	SVK	1	0	0	1
GEO	1	0	0	0	SVN	1	0	0	0
DEU	1	1	1	1	ZAF	1	1	1	1
GHA	1	0	1	1	ESP	1	0	1	1
GRC	1	0	1	1	LKA	1	0	1	1
GUY	1	1	1	1	SWZ	1	0	1	1
HTI	1	0	0	0	SWE	1	1	1	1
HND	1	0	0	0	CHE	1	0	1	1
HKG	1	0	0	1	TWN	1	1	1	1
HUN	1	0	0	1	TZA	1	0	0	1
ISL	1	0	0	1	THA	1	0	1	0
IND	1	0	0	1	TTO	1	1	1	1
IDN	1	0	0	0	TUN	1	0	0	1
IRQ	1	0	0	0	TUR	1	0	1	1
IRL	1	1	1	1	UGA	1	0	1	1
ISR	1	0	0	1	GBR	1	1	1	1
ITA	1	1	1	1	USA	1	1	1	1
JAM	1	1	1	1	URY	1	0	0	1
JPN	1	1	1	1	VEN	1	0	0	0
JOR	1	0	0	0	VNM	1	0	0	0
KAZ	1	0	0	1	ZMB	1	0	1	1
KEN	1	1	1	1	ZWE	1	1	1	1

Table 4: The countries that get included in the various data period subsamples. One means it is included, zero otherwise. For example, countries with ones in the column for 1975 means these were the countries that had non-missing interest rate data in Jan 1975

	rate				vol				nobs
	min	max	mean	std	min	max	mean	std	
ALB	5.05	38.23	12.32	8.32	0.19	4.34	0.70	0.68	183
DZA	0.09	10.13	3.72	3.75	0.42	1.70	0.57	0.26	135
AGO	2.80	134.00	55.13	47.86	1.04	25.02	6.10	5.76	110
ARG	1.10	59.77	10.02	8.81	-	-	-	-	22
ARM	3.24	80.42	22.85	20.02	0.42	17.20	3.57	3.85	170
AUS	0.75	19.40	6.30	4.19	0.08	2.72	0.34	0.41	717
AUT	3.61	10.38	6.63	1.66	0.10	0.81	0.14	0.06	373
AZE	3.92	22.10	11.76	4.37	0.97	3.03	1.54	0.46	144
BHS	0.06	9.90	4.44	2.56	0.28	1.73	0.55	0.26	463
BHR	0.69	9.98	4.73	2.18	0.14	0.52	0.24	0.08	269
BGD	1.86	11.50	8.03	1.80	0.10	1.07	0.19	0.17	205
BRB	0.24	16.02	5.88	2.42	0.17	1.75	0.37	0.25	513
BEL	0.34	14.03	6.18	2.83	0.12	1.67	0.25	0.16	718
BLZ	3.22	14.46	6.38	3.06	0.03	2.27	0.18	0.32	370
BOL	0.75	26.60	11.69	5.69	0.85	3.87	1.18	0.45	187
BWA	8.16	14.31	12.39	0.98	0.19	0.79	0.28	0.11	165
BRA	8.65	933.60	68.71	115.38	1.42	185.95	10.15	21.50	295
BGR	2.12	1232.75	48.10	131.98	6.08	1006.04	24.15	82.10	194
BDI	6.41	19.84	9.85	3.05	0.11	1.17	0.47	0.30	66
CAN	0.20	20.90	5.69	3.73	0.11	1.77	0.38	0.28	719
CPV	2.00	11.08	5.69	2.29	0.14	1.51	0.43	0.32	142
CAF	2.08	3.73	2.63	0.57	-	-	-	-	12
CHL	0.46	19.17	6.73	4.49	0.24	4.55	0.86	0.77	149
CHN	1.21	4.50	2.63	0.91	0.31	1.28	0.39	0.16	96
COL	4.35	52.64	13.75	9.33	0.26	6.38	1.33	1.55	108
CRI	3.33	24.50	15.44	4.90	1.10	5.50	1.37	0.56	163
HRV	1.90	7.60	4.27	1.62	0.31	1.03	0.50	0.17	107
CYP	2.46	6.23	5.30	0.84	0.02	1.04	0.07	0.11	400
CZE	1.66	15.54	5.75	3.71	0.16	1.52	0.31	0.22	193
DNK	2.00	20.70	9.94	5.90	0.40	4.62	0.60	0.48	380
EGY	5.26	19.40	10.53	3.26	0.03	3.79	0.44	0.53	228
SLV	2.82	6.99	3.77	0.95	-	-	-	-	56
ETH	0.04	12.00	3.83	3.71	0.02	6.74	0.35	0.71	284
EUR	0.34	11.75	5.81	3.03	0.07	1.10	0.27	0.19	311
FJI	0.07	18.65	4.06	2.74	0.05	5.47	0.67	0.81	417
FRA	0.37	18.92	6.31	3.71	0.08	1.77	0.36	0.26	600
GEO	9.95	58.44	31.79	14.05	-	-	-	-	55
DEU	0.34	12.05	4.39	2.02	0.15	1.55	0.32	0.17	683
GHA	9.38	46.75	22.22	10.27	0.53	7.69	1.05	0.93	376
GRC	0.72	25.50	11.44	6.13	0.18	6.55	0.44	0.60	359
GUY	2.84	33.75	10.67	6.91	0.04	12.99	0.44	0.95	420
HTI	4.00	27.83	16.42	7.36	1.53	6.26	2.07	0.90	157
HND	13.97	18.00	14.71	1.31	-	-	-	-	26
HKG	-0.08	12.24	3.53	2.27	0.27	3.14	0.56	0.45	223
HUN	5.55	35.30	16.81	8.95	0.52	2.90	1.01	0.47	251
ISL	4.44	34.30	11.46	6.53	0.27	5.01	1.01	0.93	270
IND	3.39	14.00	8.19	2.53	0.57	1.71	0.81	0.25	204
IDN	3.50	14.50	8.97	2.42	-	-	-	-	48
IRQ	1.20	22.00	11.63	6.32	2.16	3.91	2.56	0.46	70
IRL	0.31	39.94	8.07	4.60	0.32	15.05	0.88	1.27	480
ISR	0.29	17.96	8.73	4.54	0.23	1.58	0.58	0.28	215
ITA	0.31	22.08	7.48	5.19	0.05	2.44	0.43	0.42	718
JAM	1.75	51.98	12.81	10.16	0.23	6.48	0.97	1.11	682
JPN	0.00	8.27	3.78	2.50	0.03	0.64	0.14	0.13	600
JOR	2.05	6.88	4.93	1.71	0.23	0.80	0.35	0.12	71
KAZ	2.09	318.78	28.70	60.71	0.17	29.70	2.48	4.87	188
KEN	0.11	70.64	11.98	9.39	0.43	8.72	1.39	1.36	405

Table 5: Descriptive statistics of GFD interest rates and the computed volatility. The volatility is based on the entire interest rate data available for that country. The last column (nobs) shows the number of consecutive months with non-missing data.

	rate				volatility				nobs
	min	max	mean	std	min	max	mean	std	
KOR	2.48	19.20	9.78	4.78	0.21	1.56	0.61	0.33	275
KWT	0.60	8.87	6.12	1.78	0.03	1.19	0.18	0.19	311
KGZ	3.47	216.50	28.94	36.62	1.79	33.82	6.19	5.79	190
LVA	2.30	33.98	8.11	8.06	0.50	4.67	0.94	0.74	173
LBN	2.54	34.18	12.34	5.72	0.12	7.86	0.66	0.95	382
LTU	1.96	37.00	8.81	8.46	0.21	1.09	0.42	0.25	115
MKD	4.66	18.00	8.63	2.72	0.33	2.47	0.67	0.43	143
MDG	3.92	24.04	12.49	5.01	0.53	3.64	0.85	0.57	111
MYS	1.82	9.98	4.39	1.35	0.04	1.48	0.18	0.18	588
MLT	1.46	5.49	4.26	0.81	0.04	0.88	0.10	0.11	264
MUS	3.68	12.91	8.50	2.32	0.26	1.71	0.51	0.24	155
MEX	4.56	153.91	29.03	25.98	0.46	13.42	2.39	2.35	251
MDA	1.21	74.30	19.73	13.39	1.99	6.98	2.94	1.25	176
MNG	5.56	7.91	6.93	0.77	-	-	-	-	27
MNE	0.45	10.80	2.65	3.59	0.17	2.65	0.65	0.58	64
MAR	3.24	3.70	3.46	0.16	-	-	-	-	21
MOZ	7.15	31.65	16.70	6.53	1.53	8.99	1.93	0.93	117
NAM	6.66	21.68	11.60	3.22	0.35	1.79	0.54	0.23	217
NPL	0.62	12.88	5.48	2.62	0.03	1.76	0.50	0.40	332
NLD	0.23	13.80	4.36	2.63	0.10	1.61	0.34	0.25	718
NZL	2.62	27.20	9.51	4.74	0.11	3.92	0.63	0.67	380
NIC	1.02	6.50	3.91	1.83	-	-	-	-	55
NGA	2.00	27.50	12.95	5.42	0.00	8.69	0.79	0.94	220
NOR	0.10	15.75	7.09	3.85	0.30	9.93	0.70	0.99	312
PAK	1.21	17.42	10.05	3.80	0.21	1.79	0.59	0.30	226
PHL	2.92	43.39	12.50	6.57	0.26	5.91	1.05	0.96	399
POL	3.90	49.02	16.57	12.10	0.16	7.64	1.06	1.31	224
PRT	0.31	22.19	7.54	5.07	0.15	1.16	0.38	0.22	292
ROU	7.82	179.94	48.65	32.35	5.11	39.99	9.25	6.77	140
RUS	0.26	355.80	33.46	59.80	1.53	3.00	1.70	0.21	114
RWA	5.24	12.85	9.57	1.80	0.24	2.21	0.57	0.35	128
SAU	1.12	7.16	4.41	1.65	0.16	0.75	0.29	0.12	207
SRB	4.20	99.25	17.52	12.85	0.78	5.92	1.57	0.97	108
SLE	3.80	95.20	16.56	14.98	0.34	23.59	1.46	2.26	519
SGP	0.20	4.90	1.90	1.17	0.16	1.30	0.46	0.22	265
SVK	1.95	26.00	9.36	5.90	0.15	5.81	1.17	1.12	180
SVN	0.55	12.70	6.37	3.30	0.08	1.47	0.47	0.27	96
ZAF	1.00	22.15	8.06	5.11	0.08	1.91	0.34	0.31	719
ESP	0.31	15.27	7.39	4.40	0.15	1.45	0.36	0.24	329
LKA	6.54	21.30	13.08	3.20	0.40	3.07	0.94	0.49	345
SWZ	4.93	19.50	10.83	3.27	0.53	1.90	0.77	0.26	293
SWE	0.13	18.00	6.46	3.78	0.25	3.70	0.53	0.39	660
CHE	0.00	9.30	3.30	2.43	0.09	1.05	0.36	0.22	358
TWN	0.17	14.99	5.46	3.18	0.08	2.52	0.27	0.32	428
TZA	2.60	62.30	13.25	11.01	0.69	11.80	2.36	1.90	189
THA	1.02	19.32	6.19	4.12	0.10	3.28	0.40	0.59	156
TTO	2.30	12.11	5.92	2.52	0.04	1.44	0.18	0.22	535
TUN	4.02	11.62	7.38	2.17	0.45	0.93	0.46	0.05	239
TUR	7.92	159.44	56.71	32.63	1.07	25.30	7.21	5.85	170
UGA	2.97	43.50	16.74	11.90	0.86	10.88	1.73	1.34	358
GBR	0.42	16.27	6.77	3.58	0.18	1.29	0.48	0.28	719
USA	0.01	15.52	4.77	2.87	0.13	1.97	0.37	0.29	719
URY	2.26	146.47	26.98	25.14	1.12	9.09	2.96	2.20	87
VEN	8.89	57.05	21.98	10.40	2.58	13.77	3.60	1.70	86
VNM	3.34	15.60	7.06	2.67	0.21	4.49	0.54	0.62	147
ZMB	4.38	181.78	26.51	26.56	1.14	36.66	3.10	4.38	382
ZWE	3.05	525.00	34.97	78.92	1.60	259.00	5.55	18.79	566

Table 6: Descriptive statistics of GFD interest rates and the computed volatility. The volatility is based on the entire interest rate data available for that country. The last column (nobs) shows the number of consecutive months with non-missing data.

	1	2	3	4	5	Total
Spot Change: Δs	-1.028 (13.03)	-0.184 (13.83)	-1.845 (14.32)	-3.797 (13.85)	-2.935 (46.44)	-1.955 (24.15)
$i^* - i$	-1.594 (2.004)	0.0685 (1.882)	1.724 (2.231)	3.823 (3.326)	9.282 (8.661)	2.660 (5.820)
Excess Return: rx	-2.713 (13.70)	-0.143 (14.03)	-0.207 (14.33)	-0.0346 (13.68)	6.667 (46.56)	0.716 (24.48)
Volatility: $\sigma(i^*)$	0.301 (0.134)	0.308 (0.146)	0.426 (0.212)	0.548 (0.337)	1.079 (0.979)	0.533 (0.560)

Table 7: The whole sample of countries. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs , the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx , and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 4.

	1	2	3	4	5	Total
Spot Change: Δs	-0.805 (11.85)	0.627 (14.42)	-0.776 (19.64)	-5.063 (19.66)	-9.395 (21.48)	-3.082 (18.15)
$i^* - i$	-2.292 (2.416)	-0.570 (2.198)	1.082 (2.024)	3.438 (2.446)	11.60 (7.739)	2.653 (6.304)
Excess Return: rx	-3.241 (12.47)	0.0177 (14.80)	0.244 (19.86)	-1.677 (19.62)	2.506 (21.24)	-0.430 (18.00)
Volatility: $\sigma(i^*)$	0.303 (0.158)	0.305 (0.142)	0.415 (0.260)	0.511 (0.383)	1.372 (0.927)	0.581 (0.621)

Table 8: The sample of countries that had interest rate data on January 1975. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs , the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx , and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 4.

	1	2	3	4	5	Total
Spot Change: Δs	-0.443 (11.75)	1.060 (17.22)	0.0293 (16.03)	-4.578 (13.65)	-6.897 (54.49)	-2.166 (27.87)
$i^* - i$	-0.964 (1.609)	0.725 (1.764)	2.394 (2.165)	5.551 (2.630)	17.18 (7.755)	4.977 (7.571)
Excess Return: rx	-1.555 (12.11)	1.753 (17.26)	2.385 (15.92)	0.926 (13.35)	10.66 (53.12)	2.834 (27.47)
Volatility: $\sigma(i^*)$	0.321 (0.139)	0.308 (0.131)	0.401 (0.225)	0.684 (0.397)	1.912 (1.050)	0.725 (0.800)

Table 9: The sample of countries that had interest rate data on January 1987. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs , the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx , and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 4.

	1	2	3	4	5	Total
Spot Change: Δs	-0.861 (11.84)	1.839 (14.77)	-0.740 (13.07)	-4.033 (13.27)	-0.234 (41.09)	-0.806 (21.91)
$i^* - i$	-1.144 (1.172)	0.408 (1.040)	2.171 (1.229)	5.311 (1.928)	13.56 (4.855)	4.062 (5.781)
Excess Return: rx	-2.152 (12.14)	2.220 (14.80)	1.373 (12.92)	1.218 (12.82)	13.80 (40.06)	3.292 (22.10)
Volatility: $\sigma(i^*)$	0.278 (0.0753)	0.288 (0.0918)	0.433 (0.119)	0.645 (0.187)	1.371 (0.419)	0.603 (0.461)

Table 10: The sample countries that had interest rate data on January 1995. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs , the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx , and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 4.

Figure 3
U.S. and Australian Interest Rates: Volatility and Spread

GARCH estimates of U.S. and Australian interest rate volatility appear in the upper left panel. Interest rates appear in the upper-right panel. U.S. data are the blue dashed-lines and Australian data are the green solid lines. The lower panel plots the differences, U.S. minus Australia from the two top panels. The red dashed line is the volatility difference and the the black solid line is the interest rate differential, divided by 5. Lognormal models predict that, in the lower panel, the lines appear on opposite sides of zero, and are negatively correlated. By-and-large, the opposite seems to be true. The correlation is 0.52. Data source: 3-month interbank deposits from Global Financial Data (www.globalfinancialdata.com).

Figure 4
Average Volatility of Currency Portfolios

GFD interest rates, the whole sample.

GFD interest rates, only the countries that had non missing data on January 1975.
See tables 4, 5, and 6 to see which countries are in this sample.

GFD interest rates, only the countries that had non missing data on January 1987 Jan. See tables 4, 5, and 6 to see which countries are in this sample.

GFD interest rates, only the countries that had non missing data on January 1995 Jan. See tables 4, 5, and 6 to see which countries are in this sample.

Figure 5
Bilateral Currency Pairs

GFD interest rates: currency pairs

Figure 6
Data Source Comparison: GFD Versus FT/ICAP

GFD interest rates, the same set of countries (and years) that is in the FT/ICAP interest rate data source in below figure.

FT/ICAP interest rates, the whole sample.

The difference in volatilities between GFD and FT/ICAP interest rates (especially in early 1980's) is due to how different the GFD and FT/ICAP interest rates look even for the same country and the same date. Good examples were BEL, FRA, and ITA. So in figures 9 and 10, we replot above two figures without BEL, FRA, and ITA, after which the average volatility figure looks similar (in scale) between GFD and FT/ICAP.

GFD interest rates, same as in figure 7 but without BEL, FRA, ITA

FT/ICAP interest rates, same as in figure 8 but without BEL, FRA, ITA

Figure 7
Implied Volatility of Options on Interest Rate Futures