Interpreting the Forward Premium Anomaly

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Interpreting the forward premium anomaly

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Abstract. One of the central issues in international finance concerns the forward premium anomaly: changes in spot exchange rates are inversely related to the premium of forward rates over spot rates. We construct a numerical example of a theoretical economy with this property and discuss its potential as an explanation of the anomaly.

Une interprétation du comportement anomale de la prime à terme. L’un des problèmes importants en finance internationale est le comportement anomale de la prime à terme: les changements dans les taux de change au comptant sont inversement reliés à la prime des taux à terme sur les taux au comptant. Les auteurs construisent un exemple numérique d’une économie théorique qui possède cette propriété et discutent ses potentialités comme source d’explication de l’anomalie.

I. INTRODUCTION

Perhaps the most puzzling feature of forward exchange rates is their relation to subsequent changes in spot rates. Dozens, possibly hundreds, of studies report estimates of regressions like

\[ s_{t+1} - s_t = a_1 + b_1(f_t - s_t) + \text{residual}, \]  

where \( s_t \) is the logarithm of the domestic currency price of one unit of foreign currency in the spot market and \( f_t \) is the logarithm of the price in the forward market. Under the expectations hypothesis, forward rates are market predictions of future spot rates and we would expect estimates of \( b_1 \) to be near one. The reasoning

Two of us owe much of our interest and knowledge of international economics to Doug Purvis. We wish he were now to challenge and encourage us, as he did so often in the past. We thank Martin Evans and Stan Zin for useful comments on an earlier draft. Backus thanks the National Science Foundation for financial support.
starts with covered interest parity: \( f_t - s_t = r_t - r_t^* \), with \( r \) and \( r^* \) the domestic and foreign interest rates with the same maturity as the forward contract. Under the expectations hypothesis, a weak currency (one whose spot rate is expected to rise) requires a high interest rate in compensation, and hence a high forward price. In fact, we generally see the opposite. Estimates of \( b_1 \) are invariably negative for major currencies, implying that a high interest rate currency tends to appreciate (its spot rate falls), not depreciate. Prominent studies reporting similar regressions include Bilson (1981), Fama (1984), Froot and Frankel (1989), and especially Hodrick’s (1987) exhaustive survey.

We refer to this feature of the data as the forward premium anomaly. After reviewing the evidence, we describe a theoretical economy with the same property. The structure is a cousin of Hamilton’s (1989) two-regime model, in which the stochastic process for the spot rate and the pricing kernel switches between two alternatives. We conclude by speculating on ways in which the properties of the pricing kernel in our example might be tied to observed properties of asset prices.

II. EVIDENCE

A large body of statistical work has established that changes in spot exchange rates are negatively correlated with the forward premium. Hodrick’s (1987) monograph contains a comprehensive list of early references, and Canova and Marrinan (1993) and Mark, Wu, and Hai (1993) review some of the more recent contributions. Taken as a whole, these studies find that estimates of the slope coefficient \( b_1 \) in equation (1) are negative for exchange rates between most major currencies for a number of different subperiods. Examples for five currencies versus the U.S. dollar are reported in table 1, where \( b_1 \) ranges from \(-0.81\) for the franc to \(-3.54\) for the Deutschmark. Only the estimate for the franc is within two standard errors of one.

This evidence conflicts with the expectations hypothesis of the forward rate. Under the log-linear version of the expectations hypothesis, the forward rate is the market’s prediction of the future spot rate: \( f_t = E_t s_{t+1} \). The forward premium, under this hypothesis, is the expected rate of depreciation, which we label \( q_t \):

\[
f_t - s_t = E_t s_{t+1} - s_t \equiv q_t.
\]

This implies, obviously, that \( b_1 = 1 \). In the data, \( b_1 \) is clearly not one. In an effort to understand this discrepancy between theory and data, Fama (1984, section 2) suggests that we include in the forward premium a factor \( p \), which he terms a risk premium:

\[
f_t - s_t = (f_t - E_t s_{t+1}) + (E_t s_{t+1} - s_t)
\]

\[
= p_t + q_t.
\]

Given a ‘risk premium’ with the right properties, we can presumably account for the forward premium anomaly.
TABLE 1
Spot exchange rate regressions

<table>
<thead>
<tr>
<th>Currency</th>
<th>$a_1$</th>
<th>$b_1$</th>
<th>Standard error</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>British pound</td>
<td>$-0.0067$</td>
<td>$-2.306$</td>
<td>$0.0322$</td>
<td>$0.0344$</td>
</tr>
<tr>
<td></td>
<td>$(0.0028)$</td>
<td>$(0.862)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canadian dollar</td>
<td>$-0.0027$</td>
<td>$-1.464$</td>
<td>$0.0120$</td>
<td>$0.0247$</td>
</tr>
<tr>
<td></td>
<td>$(0.0009)$</td>
<td>$(0.581)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>French franc</td>
<td>$-0.0026$</td>
<td>$-0.806$</td>
<td>$0.0326$</td>
<td>$0.0015$</td>
</tr>
<tr>
<td></td>
<td>$(0.0032)$</td>
<td>$(0.928)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>German mark</td>
<td>$0.0032$</td>
<td>$-3.542$</td>
<td>$0.0333$</td>
<td>$0.0287$</td>
</tr>
<tr>
<td></td>
<td>$(0.0043)$</td>
<td>$(1.348)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japanese yen</td>
<td>$0.0084$</td>
<td>$-1.813$</td>
<td>$0.0334$</td>
<td>$0.0201$</td>
</tr>
<tr>
<td></td>
<td>$(0.0032)$</td>
<td>$(0.719)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTES
The table reports statistics from regressions of the form

$$s_{t+1} - s_t = a_1 + b_1 (f_t - s_t) + \text{residual},$$

where $s$ and $f$ are logarithms of spot and forward exchange rates, respectively, measured as dollars per unit of foreign currency. The data are last Friday of the month, from the Harris Bank’s Weekly Review: International Money Markets and Foreign Exchange, compiled by Richard Levich at New York University’s Stern School of Business. Dates $t$ run from July 1974 to April 1990. Numbers in parentheses are Newey-West standard errors and Standard error is the estimated standard deviation of the residual.

But what are these properties of the risk premium? One necessary condition is that it have positive variance. If the risk premium were constant, then it would appear in the regression intercept $a_1$ and have no effect on the estimated slope coefficient. A second necessary condition is that the rate of depreciation have a predictable component. Although most of the variance of exchange rate changes seems to be unpredictable, estimates of equation (1), even with its puzzling coefficient, indicate that part of the change in the spot rate can be predicted by the forward premium. The $R^2$s, for example, are in the neighbourhood of 0.02, which is small but significant, both economically and statistically.

Two other conditions build on the first two: the two components of the forward premium must be negatively correlated, and the variance of the risk premium $p$ must be greater than the variance of the expected rate of depreciation $q$. Using the decomposition (2), the spot rate regression can be written:

$$s_{t+1} - s_t = a_1 + b_1 (p_t + q_t) + \text{residual}.$$

The population regression line thus has slope

$$b_1 = \text{Cov} (q, p + q)/\text{Var} (p + q) = [\text{Cov} (q, p) + \text{Var} (q)]/\text{Var} (p + q). \quad (3)$$
Clearly, if $\text{Var}(p) = 0$, the slope is one, so Property One is required. More than that, for the slope to be negative, the covariance of $p$ and $q$ must be negative and larger in absolute value than the variance of $q$. This implies, since correlations are less than one in absolute value, that the variance of the risk premium $p$ is greater than the variance of the expected rate of depreciation $q$. This implication was first noted by Fama (1984, 327).

We can get some idea of the magnitudes involved by looking more closely at the data. We see in table 1 that the regression slope for the dollar/pound exchange rate, to take one example, is $b_1 = -2.306$, a typical negative value. Although we do not know the expected depreciation rates used by market participants, we can use the regression’s fitted values to approximate them. The standard deviation of these fitted values is 0.006523 (about 0.65 per cent per month); see table 2. By similar means, we might estimate the standard deviation of the risk premium with fitted values from the complementary regression,

$$f_t - s_{t+1} = a_2 + b_2(f_t - s_t) + \text{residual}.$$ 

The slope for this regression is $b_2 = 1 - b_1 = +3.306$. The standard deviation in this case is 0.009353 (also reported in table 2). Since $b_1$ and $b_2$ have opposite signs, our estimates of the two components of the forward premium have a correlation of minus one. That fact allows us to compute the moments in the regression slope formula, equation (3):

$$\text{Var}(p) = (0.009353)^2 = 0.8748 \times 10^{-4}$$

$$\text{Var}(q) = (0.006523)^2 = 0.4255 \times 10^{-4}$$

$$\text{Cov}(p, q) = -(0.009353)(0.006523) = -0.6101 \times 10^{-4}$$

$$\text{Var}(p + q) = 0.8009 \times 10^{-5}$$

$$b_1 = -2.306.$$ 

These numbers work out simply as a feature of least squares, but they give us a rough idea of the magnitudes involved in our three properties: the variances of $p$ and $q$ and the covariance between these two components. Since fitted values are projections on a limited information set, rather than conditional moments, we would expect them to be less variable than the conditional means, $q_t = E_t s_{t+1} - s_t$ and $p_t = f_t - E_t s_{t+1}$. The population variances, then, are likely to be larger than we have estimated, and the population correlation is likely to be smaller in absolute value.

At the level presented here, the evidence for time-varying risk premiums is simply a way of labelling the puzzling relation between spot and forward exchange rates. Until we have a theory of a risk premium with these properties, the evidence of highly variable $p$ is simply a measure of what remains to be explained. In the following pages we describe the properties a theory must have to produce risk premiums of this sort and construct a numerical example that has them.
TABLE 2
Estimated standard deviations of forward premium components

<table>
<thead>
<tr>
<th>Currency</th>
<th>Estimated standard deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>risk premium $p$</td>
</tr>
<tr>
<td>British pound</td>
<td>0.00935</td>
</tr>
<tr>
<td>Canadian dollar</td>
<td>0.00355</td>
</tr>
<tr>
<td>French franc</td>
<td>0.00602</td>
</tr>
<tr>
<td>German mark</td>
<td>0.00798</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>0.00833</td>
</tr>
</tbody>
</table>

NOTES
The forward premium can be decomposed into

$$f_t - s_t = (f_t - E_{t} S_{t+1}) + (E_{t} S_{t+1} - s_t) = p_t + q_t,$$

where $p$ is referred to as a risk premium and $q$ as the expected rate of depreciation. The table reports estimated standard deviations of fitted values of the regression in Table 1 and its complement,

$$s_{t+1} - f_t = a_2 + b_2 (f_t - s_t) - \text{residual}.$$

III. A THEORETICAL FRAMEWORK

For more than a decade, economists and others have suggested theoretical structures that might account for the forward premium anomaly. One of the first to go beyond the expectations hypothesis was Harris and Purvis (1982), who examined a macroeconomic model of a small open economy in which agents have incomplete information about current policy. As a result of this information structure, the forward rate is not generally the expectation of the future spot rate. We take a somewhat different approach. As is done in most of financial economics, we disregard the macroeconomic fundamentals underlying exchange rate movements and focus instead on the restrictions placed on asset prices by the absence of pure arbitrage opportunities. In such arbitrage-free economies, the price of an asset can be thought of as the value of its underlying state-contingent claims. For this reason our theoretical framework consists of a pricing relation that connects the forward exchange rate to the stochastic process for state-contingent claims prices and the rate of depreciation of the spot rate, and thus to regressions like equation (1). The question we ask is what this stochastic process must look like to account for the forward premium anomaly. A deeper question, which we do not address, is what economic primitives might give rise to such a process.

Our starting point for asset pricing is what Sargent (1987) calls the pricing kernel: the stochastic process that governs prices of state-contingent claims. The kernel is the building block of theories of fixed-income securities, in the sense that a description of the kernel is sufficient to characterize prices of risk-free bonds and
related derivative assets. More generally, if the pricing kernel is \( m \), then excess returns \( x \) satisfy the relation,

\[
0 = E_t(m_{t+1} x_{t+1}),
\]

where \( E_t \) is the expectation conditional on the date-\( t \) information set and \( x \) is the return on a zero investment portfolio. In representative agent settings the pricing kernel is the nominal intertemporal marginal rate of substitution and the pricing relation (4) is one of the agent’s first-order conditions.

We apply the pricing relation to foreign currencies by considering an investment strategy of buying \( 1/S_t \) units of foreign currency forward and converting the proceeds back to domestic currency in the next period at the spot exchange rate. This strategy requires no initial investment; so it is a balanced portfolio strategy. Its return is

\[
x_{t+1} = (S_{t+1} - F_t)/S_t,
\]

where \( S \) and \( F \) are levels (rather than logarithms) of spot and forward exchange rates. Equation (4) then implies

\[
(E_t m_{t+1})(F_t/S_t) = E_t(d_{t+1} m_{t+1}),
\]

where \( d_{t+1} = S_{t+1}/S_t \) is (one plus) the rate of depreciation of the domestic currency. Since the expectation of a product is the product of the expectations plus the covariance, the equilibrium forward rate satisfies

\[
F_t/S_t = E_t d_{t+1} + \text{Cov}_t(m_{t+1}, d_{t+1})/E_t m_{t+1}.
\]

Thus, the conditional covariance between the pricing kernel and the rate of depreciation determines the relation, in the theory, between the forward rate and the expected future spot rate. If the covariance is non-zero, the forward rate need not equal the expected future spot rate.

We breathe some life into the theory by putting some structure on the stochastic process for the pricing kernel \( m \) and the rate of depreciation \( d \). A convenient choice is a process that is conditionally log-normal, as in Hansen and Hodrick (1983, section 4.3). The log-normal distribution has two useful properties that allow us to express the theory in the same form as the regression evidence. The first property is: if a random variable \( x \) is log-normal, so that \( \log x \) is normal with mean \( \mu_x \) and variance \( \sigma_x^2 \), then \( E(x) = \exp(\mu_x + \sigma_x^2/2) \). The second property follows from the first: if \( (\log x, \log y) \) is bivariate normal with mean \( (\mu_x, \mu_y) \) and variance \( \Sigma \), then \( E(xy) = (Ex)(Ey) \exp(\sigma_{xy}) \), where \( \sigma_{xy} \) is the covariance between the logarithms of \( x \) and \( y \). Property two applied to equation (5) implies

\[
F_t/S_t = \exp(\sigma_{md}x)E_t d_{t+1},
\]
where $\sigma_{md,t}$ is the conditional covariance at date $t$ between the logarithms of the pricing kernel and the rate of depreciation.

Equation (7) characterizes the properties of the risk premium and the forward premium regression in the theory. If $\sigma_{md} = 0$, the expectations hypothesis holds exactly: the forward price $F_t$ equals the expected future spot price $E_t S_{t+1}$. Even with non-zero values and the non-linearity induced by logarithms, the theory has no hope of resolving the forward premium anomaly if the conditional variance matrix $\Sigma$ is constant. By Property One of log-normal random variables,

$$\log E_t d_{t+1} = E_t s_{t+1} - s_t + \sigma_{dd,t}/2,$$

and the forward premium can be expressed

$$f_t - s_t = (\sigma_{md,t} + \sigma_{dd,t}/2) + (E_t s_{t+1} - s_t).$$

This is the theoretical analogue of Fama’s decomposition, equation (2) of the last section, and defines the risk premium by

$$p_t = \sigma_{md,t} + \sigma_{dd,t}/2.$$

The risk premium is constant if the matrix $\Sigma$ of second moments is constant, so the theory with constant second moments does not deliver the time-varying risk premiums that we seem to see in the data. If we turn equation (9) around, we get the forward premium regression:

$$E_t s_{t+1} - s_t = -(\sigma_{md,t} + \sigma_{dd,t}/2) + (f_t - s_t).$$

With constant second moments, $\sigma_{md}$ and $\sigma_{dd}$, the forward rate regression produces a slope coefficient $b_1 = 1$, not the strongly negative slope we generally see in the data. All of these results are independent of any relation between the conditional means of the logarithms of $m$ and $d$.

In short, the log-normal model with constant second moments produces a risk premium that drives forward rates away from expected future spot rates, but not one that accounts for the forward premium anomaly. Hodrick and Hansen (1983, section 4.3) pointed this fact out some years ago. What we need, apparently, is an extension that allows the risk premium on forward contracts to vary through time, as it seems to do in the data. From equations (6) and (11), we see that we need to modify the process to permit the conditional covariance between the pricing kernel and the rate of depreciation to vary through time. In the next section, we construct an example in which this covariance alternates between positive and negative values.

IV. AN EXAMPLE

We have seen that to account for the forward premium anomaly the theory requires a stochastic process for the pricing kernel and the rate of depreciation in which their
conditional variance exhibits considerable variation. A relatively straightforward way of providing such a process involves a variant of Hamilton’s (1989) ‘regime’ model. Versions of this model have been applied to currency prices by, among others, Engel and Hamilton (1990) and Evans and Lewis (1993).

Before turning to a specific example, it is worth reviewing our problem. A necessary condition for the theory to account for the forward premium anomaly is that the risk premium vary through time, which suggests, in the theory, substantial variation in the conditional covariance between the pricing kernel \( m \) and the rate of depreciation \( d \). An example might be something like equation (11) with a conditional covariance \( \sigma_{md} \) that changes through time. We shall need more to mimic the forward premium anomaly, but it is a move in the right direction.

Consider, then, a model in which the conditional variance matrix \( \Sigma \) for logarithms of the pricing kernel and the depreciation rate alternates between two values. Let us say, to be specific, that there are two ‘regimes,’ labelled 1 and 2, whose transitions are governed by the probabilities \( \pi_{ij} \) for \( i, j = 1, 2 \). If at date \( t \) we are in regime \( i \), then \( m_{t+1} \) and \( d_{t+1} \) are conditionally log-normal with conditional variance matrix \( \Sigma^i \). Equation (11) of the last section tells us that in regime \( i \) the expected rate of depreciation is

\[
E_t s_{t+1} - s_t = -(\sigma_{md} + \sigma_{dd}^i / 2) + (f_t - s_t).
\]  

Since second moments change through time, the model is capable, in principle, of generating slope coefficients \( b_1 \) that differ from 1.

This mechanism, by itself, is not enough to generate a negative slope coefficient. Suppose, for example, that second moments alternate independent of the expected rate of depreciation — say \( E_t s_{t+1} - s_t \) in the log-normal version. Then the two components of Fama’s decomposition, the expected rate of depreciation and the risk premium, are uncorrelated. In a regression of changes in the spot rate, \( s_{t+1} \), on the forward premium, \( f_t - s_t \), variations in the risk premium act like random measurement error and drive the slope coefficient towards zero. They do not, however, produce a negative slope coefficient. We need, in addition to a changing second moments, a connection with the expected rate of depreciation.

With that requirement in mind, consider an extension in which the conditional mean of the spot rate, as well as conditional variances and covariances, vary across the two regimes. A relatively simple example includes:

\[
\log m_{t+1} = \mu + \epsilon^m_{t+1}
\]

\[
\log d_{t+1} = \delta + \epsilon^d_{t+1},
\]

with \( \text{Cov}_t (\epsilon^m_{t+1}, \epsilon^d_{t+1}) = \Sigma^i \) in regime \( i \). In regime \( i \) the two components of Fama’s decomposition are then

\[
q_t = E_t s_{t+1} - s_t = \delta^i
\]

\[
p_t = \sigma_{md} + \sigma_{dd}^i / 2.
\]
Since the conditional mean, $\delta^i$, of the depreciation rate varies across regimes, there can be a connection between the expected rate of depreciation and the risk premium. Depending on how we choose $\delta$ and $\Sigma$ in the two regimes, we can make them positively or negatively correlated.

A numerical example shows how we can use this structure to replicate the negative slope coefficient that we examined in section II. Let us say that the risk premium $p$ alternates between two values, $p^1 = g$ and $p^2 = -g$. If the Markov chain for regimes has equal unconditional probabilities for each, then the mean value of $p$ is zero and its standard deviation is $g$. Similarly, let us say that expected depreciation $q$ alternates between $q^1 = -h$ and $q^2 = h$. The forward premium is thus $g - h$ in regime 1, $h - g$ in regime 2. You can see that this two-regime example produces perfect correlation between the two components. If $g$ and $h$ have the same sign, the correlation is $-1$, a strong version of the negative correlation we saw that we needed to account for the negative slope coefficient in the forward premium regression. The theoretical regression coefficient is

$$b_1 = -h(g - h)/(g - h)^2 = -h/(g - h).$$

As long as $g > h$, the coefficient is negative. Taking numbers from the numerical example in section II, we set $g = 0.009353$ and $h = 0.006523$, which produces, in the theory, a slope coefficient of $-2.306$, just as it did in the data. This example, in other words, reproduces the puzzling negative slope coefficient we see in the data.

The example has been rigged, of course, to resolve the forward premium anomaly, but it illustrates the properties a theory must have to do so. We can easily imagine more complex processes with similar properties. We could, for instance, have a continuum of `regimes' in which there is a negative correlation between the expected rate of depreciation and the conditional covariance, and thus eliminate the artificial two-value feature of this example.

V. DISCUSSION

The example of the previous section shows that theory is capable of accounting for the forward premium anomaly, but not that it does so in a persuasive way. We discuss some of the issues and evidence that might be considered if we are to take the example, or something like it, seriously as an explanation of the forward premium anomaly.

Conditional variances and covariances. We have seen (equation (10)) that a time-varying risk premium requires, in the theory, changes in the conditional variance of the spot exchange rate or the conditional covariance between the depreciation rate and the pricing kernel. There is no lack of evidence that the conditional variance of the exchange rate varies through time: among the many studies to make this point are Domowitz and Hakkio (1985), who modelled the variance as an ARCH process, and Engel and Hamilton (1990) and Evans and Lewis (1993), who found that the spot rate alternated between high and low variance periods.
Despite this evidence for a changing conditional variance, we think that it must be the conditional covariance, not the conditional variance, that drives the risk premium in equation (10). One reason is that the conditional variance appears in the risk premium only because we defined the expected rate of depreciation in logarithms; see equation (8). In levels, as in equation (6), the conditional variance does not enter the theoretical formula for the risk premium. Yet estimates of the levels version of equation (1),

\[(S_{t+1} - S_t)/S_t = a_1 + b_1(F_t - S_t)/S_t,\]

produce similar negative slope coefficients. See, for example, Backus, Gregory, and Telmer (1993, table 2) or Hodrick and Srivastava (1986, table 2). Apparently the conditional variance does not play a big role in generating the forward premium anomaly.

A second reason for emphasizing the conditional covariance over the variance is that attempts to tie the risk premium to the conditional variance have been notably unsuccessful. Both Domowitz and Hakkio (1985) and Bekaert and Hodrick (1993) find, using ARCH and GARCH models, that considerable variation in the risk premium remains after taking the conditional variance of the spot rate into account. Similarly, Engel and Hamilton (1990, table 1) find, in a two-regime model, that while the expected rate of depreciation and the conditional variance are negatively correlated across regimes, they do not account for the departures from the expectations hypothesis observed in their data. We might have expected this result from the magnitudes involved. The most favourable comparison for this approach is the Deutschemark. We saw in table 2 that the data suggest a standard deviation of the risk premium for the Deutschemark of about 0.00798. But the estimated difference in the conditional variance \(\sigma_{dd}\) between regimes is only 0.00075, which is smaller by more than a factor of ten. (These monthly numbers are from Evans and Lewis 1993, table ii, computed as 0.00075 = (11.313 - 3.862)/100^2.) For the pound and the yen, the difference in variance is even smaller. The conditional covariance, in contrast, is potentially much larger, since the pricing kernel is estimated to have much greater variability than the spot exchange rate. For the spot exchange rate, for example, we estimated the monthly conditional standard deviation to be 0.0333 (see table 1, again for the Deutschemark), while the pricing kernel has an estimated conditional standard deviation of about 0.4 (Backus and Zin 1994, section 6).

Our final reason for downplaying the conditional variance is an aesthetic one: basing the risk premium on the conditional variance violates symmetry and produces a theory for the foreign country opposite to that for the home country. In the Engel and Hamilton (1990) model, for example, the dollar tends to depreciate relative to the Deutschemark in the low variance regime and appreciate in the high variance regime. This difference tends to produce (see equation (10)) a negative correlation between the risk premium and the expected rate of depreciation. If we view this situation from the perspective of Germany, however, we get the opposite result: depreciation (relative to the dollar) occurs in the high variance regime. While a theory of this sort is not impossible, it does not seem likely.
For these reasons, we think that the emphasis in the theory of the risk premium must be on the conditional covariance between the pricing kernel and the rate of depreciation, to which we now turn.

To say something about the conditional covariance of the pricing kernel and the rate of depreciation, we need to be more specific about the properties of the kernel. Without this specificity, the pricing kernel is simply an arbitrary unobservable. One line of attack has been the representative-agent framework, in which the pricing kernel is the (nominal) intertemporal marginal rate of substitution. This approach effectively makes the pricing kernel observable by tying it to aggregate consumption and price level data. Within this framework and using power utility, Cumby (1988, figure 4) finds some evidence that the conditional covariance changes through time and even changes sign. The catch is that the standard errors on his estimated conditional covariances are large relative to the estimates. Other attempts have been notably less successful. Backus, Gregory, and Telmer (1993, table v) generate risk premiums in a calibrated model in which the representative agent has non-separable preferences but find that they exhibit, at best, only half as much variation as we see in the data. Moreover, the correlation of the risk premium and the expected rate of depreciation is close to zero. Bekaert, Hodrick, and Marshall (1992) depart from expected utility, but have little more success. Neither study generates negative regression coefficients.

We have argued, in short, that the theory requires substantial variation in the conditional covariance between the pricing kernel and the rate of depreciation, and that representative agent theories have not had this property to date. That leaves us, to some extent, in limbo. If we allow ourselves enough freedom in choosing a pricing kernel and its relation to the depreciation rate, we can produce a risk premium that does pretty much anything we like. Clearly we would like to do more, and we relate the required properties of the pricing kernel to observed properties of asset prices. One line of attack is to use interest rates to identify innovations to the pricing kernel. In a one-factor model of bond prices, for example, innovations in the short rate are perfectly correlated with innovations in the pricing kernel and can be used to estimate them from interest rate data; see, for example, Backus and Zin (1994). Perhaps we could adopt a similar approach to the conditional covariance between the kernel and the rate of depreciation.

VI. FINAL THOUGHTS

We have described the properties a theory must have to reproduce the puzzling inverse relation between rates of depreciation and forward premiums and constructed a numerical example that has them. Before we claim victory, however, we would like to mention two issues that might provoke scepticism about the anomaly and our proposed resolution. One is that the evidence is based on a relatively short sample and might simply be a strange draw from an economy with less exotic properties. After twenty years’ experience with floating exchange rates, this claim has less force than it once did, but it remains a possibility. A second question is whether
the example is too complex to be plausible, or at least persuasive. Certainly we need more direct evidence on the properties of the pricing kernel before our example can be regarded as more than an indication that imagination and free parameters can explain virtually anything. But if nothing else, it provides a tempting target for future work.

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