“Let this deal go down,” thinks Brad as he calls Elizabeth Steiner.

Brad Kaiser is a vice president in fixed income sales and trading at Wright Structured Products Inc. Ms. Steiner is a senior trader at Winterland Asset Management, a large hedge fund. She is a major investor in credit derivatives, but the recent decline in corporate implied default probabilities has her looking for new credit plays. Brad is trying to interest Ms Steiner in the ABS market for home equity loans (HEL).

**Background on HEL securitization.** Home equity loans are residential mortgages. Originally just second-lien residential mortgages, HELs are now any mortgage to a credit-impaired homeowner even if the mortgage is a first-lien. HELs are not eligible for FNMA, GNMA or other agency default insurance. Consequently, they are exposed to credit risk as well as interest rate risk. HELs are originated by banks and finance companies and then used as collateral for asset backed securitizations. In a typical securitization a portfolio of HELs is put into a special investment vehicle (i.e., a trust) which then issues bonds and other securities backed by the cash flows of the underlying mortgages. A servicing company collects coupon and principal payments from the mortgages, represents the trust in the event of mortgage defaults, and distributes payments to the ABS investors. The servicer is paid an annual servicing fee tied, over time, to the remaining principal of the mortgages in the trust. This fee, currently 50 basis points per annum, is deducted from the mortgage cash flows before any payments to the ABS investors.

Brad has put together a large HEL securitization identified as Trust HC-11. Exhibit 1 summarizes key features of the mortgages involved and the terms of the ABS bonds issued. HEL mortgages can have a variety of interest rate structures ranging from straight fixed and floating rate coupons to so-called hybrid payment structures. The underlying mortgages in HC-11 are all hybrids. They initially pay a fixed rate for three years and then convert into paying a floating rate indexed to 1 year Treasury. The mortgage floating rate is reset annually but has a floor equal to the initial fixed rate. There are no other caps or floors on the mortgage rate. The mortgages all have balloon payments at maturity of the full principal balance. Prior to maturity there are only interest payments, but no scheduled principal payments. There may, however, be unscheduled principal payments due to defaults and prepayments. These are immediately passed through to the bonds.

For simplicity we are summarizing the ABS bonds as a group. All of the bonds have floating rate coupons indexed to 1 year Treasury but subject to an available funds cap. For this specific Trust, this cap means simply that the total coupon payments due to the ABS bonds are contractually bounded from above by the actual interest payments received from the mortgages net of the

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1 © 2005 Duane Seppi. The persons and events in this case are fictional and are intended solely as a basis for discussion in the 2005 Financial Engineering Competition organized by the Tepper School of Business at Carnegie Mellon University and sponsored by Lehman Brothers and Appaloosa Management. This case does not necessarily reflect the views of any of the sponsors. I thank Stefano Risa and Marco Naldi for many helpful insights about the HEL market. Chris Telmer, as always, helped a lot with the case. Mark Broadie and Burton Hollifield gave comments on an early draft.

2 Lehman Brothers’ *The ABCs of HELs* has further details about home equity loans and HEL securitization.

3 In practice it is common for pools to have a mix of hybrid and fixed rate mortgages.
servicing fee. Interest on both the mortgages and on the ABS bonds is paid in arrears at the end of each year.

Brad bought the underlying HEL mortgages from the mortgage originators at 102 per 100 of principal. The ABS bonds, priced at par, sold well. Brad hopes that Ms Steiner will buy the residual payment tranche so he can get it out of his inventory and move on.

The residual tranche is an equity claim on the *excess spread* defined as the difference between the interest payments on the mortgages and the interest payments on the ABS bonds. The excess spread is also a significant source of credit enhancement for the ABS bonds. At the origination date the initial interest on the HEL mortgages is significantly above the initial coupons on the ABS bonds. This is standard given (a) the high initial interest rates on HEL mortgages relative to ABS bond coupons and (b) the Trust’s initial over collateralization. In particular, as shown in Exhibit 1, the starting mortgage principal balance is greater than the starting bond principal balance. As time passes the excess spread cash flow varies given the evolution of future interest rates, prepayments, and defaults. Given the particular available funds cap here, the excess spread can never go negative, but it can get small or go to zero. So the residual tranche is risky.

**Waterfall.** The dynamics of the excess spread and the cash flows received by the bond and residual tranches depend on the payment *waterfall*. In each period the Trust receives payments of interest from the mortgages along with early payments of principal due to voluntary prepayments and recoveries of principal from involuntary defaults. In turn, the Trust makes payments of coupons and principal to the ABS bonds and occasional disbursements of excess spread to the residual tranche. The waterfall simply described the priority rules governing which tranches get paid first. Payment waterfalls can get complicated in practice, but the one considered here is comparatively not too convoluted.

The dynamics of the payments to the residual tranche depend on the changing amount of over-collateralization – i.e., the difference between the outstanding principal balances on the underlying mortgages and the ABS bonds. First consider the mortgages. If $M_{t}$ is the outstanding mortgage principal balance in year $t$ and $M_{t-1}$ is the prior balance in year $t-1$, then the mortgage principal dynamics are

$$M_{t} = M_{t-1} - \text{[full principal from prepayments]} - \text{[full principal from defaulted mortgages]}$$

Similarly, if $B_{t}$ is the outstanding bond principal balance in year $t$ and $B_{t-1}$ is the prior bond balance in year $t-1$, then the bond principal dynamics are

$$B_{t} = \max\{0, B_{t-1} - \text{[full principal from prepayments]} - \text{[recovered principal from defaulted mortgages]} - \min\{\text{lost principal from defaults}, \text{Mortgage interest received} - \text{Bond interest due}\}\}$$

The max with 0 means the bond is retired once the remaining principal is paid off in full. You cannot have a negative principal balance on the ABS bonds. The min{lost principal from defaults, Mortgage interest received – Bond interest due} term reflects the fact that the available excess spread is first used, if needed, to offset losses from current defaults. The key difference between the mortgage and bond principal dynamics is the following: Every dollar of default loss reduces the mortgage balance $M_{t}$ by a dollar. In contrast, defaults, if the excess spread is insufficient, reduce the bond principal $B_{t}$ by less than dollar-for-dollar. When this happens, the over-
collateralization in the trust shrinks. If there are enough defaults, the ABS bonds are not paid off in full even with the excess spread.

For simplicity, we assume that positive excess spread at date $t$ cannot be used to catch up from default losses incurred prior to date $t$. The absence of a catch up provision simplifies the waterfall dynamics. Thus, in this trust, any excess spread not needed to defray losses from current defaults is disbursed to the residual tranche.\footnote{In practice, excess spread is often first directed to pay down the bond balance so as to maintain a target level of over-collateralization. This is called the turbo feature. Only when the target level of over-collateralization is reached are disbursements of excess spread to the residual tranche triggered.}

The timing and size of payments to the residual tranche are determined by the interaction of five factors: Interest compression between the mortgages and the ABS bonds, the timing of voluntary prepayments by home owners, the number and severity of mortgage defaults, and the waterfall rule described above. The resulting cash flows are a complex mix of interest rate risk, credit risk, and, in particular, real estate risk.

**Interest rate compression.** While the excess spread is initially positive and large, the fact that the ABS bond coupon is floating while the HEL mortgage rate is initially fixed means that rising Treasury rates over the first three years shrinks the excess spread. The resulting interest rate compression reduces future payment to the residual tranche. Falling Treasury rates have the opposite effect.

**Voluntary mortgage prepayment.** Homeowners have the option to prepay home equity loans after an initial 3-year freeze-out window. During the freeze-out window, prepayments are not permitted.

The most important prepayment driver once prepayment is permitted is the strong motive to refinance high interest rate HELs with either lower rate conventional mortgages or new HELs with lower rates. Rate improvement can happen several ways. Individual homeowners’ FICO credit scores change randomly, but those who do not default over time see their credit improve on average. Thus, a large number of the non-delinquent mortgages can be expected to prepay in year 4 due to credit curing over the contractual freeze-out window. From year 5 on, prepayment drops off slightly but still stays high.

Another potential driver of mortgage prepayment is reference rate refinancing. However, since these HELs switch to floating rates after the prepayment freeze, prepayments will not be sensitive to Treasury rate changes unless the floor binds. So, again for simplicity, we will ignore Treasury rate induced prepayments here.

Yet another motive for mortgage prepayment is home price appreciation (HPA). HPA increases prepayment through two channels. First, higher home prices increase the loan-to-value (LTV) ratio for the existing mortgages, thereby allowing the existing loan to be refinanced at a lower rate. In particular, even if the homeowner’s personal credit is unchanged, the collateral itself (i.e., the home) has gotten better. Second, homeowners may instead want to borrow more against their increased home equity via a new larger mortgage. Again this leads to prepayment of the old mortgage.

For both prepayments (and defaults) Wright Research measures the effects of market-wide HPA using the censored cumulative HPA return in decimal form (where 0.01 = 1 percent):
CHPA_t = \max\{ -0.05, \min\{ 0.15, \frac{H_t - H_0}{H_0} \} \}

where \( H_t \) is the FHLBB housing price index on date \( t \) and \( H_0 \) is the index on the origination date \( t_0 \). The censoring bounds the percentage HPA between –5 and +15 percent. Wright Research has found that the incremental effect of additional changes in HPA dies out beyond that range. Wright Research has also determined that the impact of HPA just depends on the magnitude of the HPA change, not the length of time over which it occurs.

what really matters is simply changes relative to the initial home valuation at origination independent of how long of a time period the change is measured over.

The annualized conditional refinancing rate (CRR) measures voluntary prepayment per year as a fraction of the outstanding mortgages. As a first pass, Wright Research estimated the following OLS regression:

\[
\text{annualized CRR}_t = 0.15 + 0.40 \text{FC}_t + 2.0 [\text{CHPA}_t - 0.03] \\
0.04 \quad 0.05 \quad 0.2
\]

SE of regression residual = .05

where

- annualized CRRt is measured in decimal form so that .01 = 1 percent
- FCt = a “first chance” dummy variable equal to 1 if date t is the first year after the end of the prepayment freeze and = 0 everywhere else.
- The CHPA effect on prepayment is not centered at 0 percent, but rather at a baseline of 3 percent.

Hint: If you simply plug numbers into the fitted CRR formula, you sometimes get negative CRRs. Does that make sense? Is there some quick and dirty fix to get around this problem?

**Default.** HEL mortgages do not qualify for FNMA or GNMA insurance. Given the impaired credit of the borrowers and high loan-to-value ratios of HEL mortgages, the danger of default is significant. Wright Research estimates that the loss severity of HEL mortgage defaults, after payment of current interest due, is a 35 percent loss of the mortgage principal balance at the time of the default. For simplicity you can assume that the 65 percent recovery is received by the Trust at the time of default.5

Default dynamics are measured by the conditional default rate (CDR). The annualized CDR is the fraction of a pool of mortgages that defaults per year. The CDRs for home equity loans are not constant over time. CDRs change as mortgages season and because of home price appreciation. First, as the pool seasons, the credit quality of the remaining mortgages tends to deteriorate. Homeowners whose credit improved enough to refinance tend to do so. Thus, the mortgages left behind in the Trust tend to be skewed towards homeowners whose credit is either unchanged or which worsened.

Wright Research has developed the following model of defaults:

5 The mortgage servicer typically advances interest payments to the ABS bondholders on mortgages that are delinquent but not in default. This effectively insulates the Trust from credit risk on current interest.
Annualized CDR_t = f(t)*[1 + g(CHPAt)]

where
- the annualized CRR is measured in decimal form so that .01 = 1 percent
- \( f(t) = (0.015)*(t - t_0) + (-0.007)*(t - t_3) \) where \( t - t_0 \) is the time elapsed at date t since origination in year \( t_0 \) and \( t - t_3 \) is the time elapsed since year 3.
- \( g(CHPAt) = -10.0*(CHPAt - 0.03) \) where 3 percent acts again as a baseline return for CHPA.

The \( f(t) \) term is simply a function of time that causes the annual rate of defaults to ramp up over the first 3 years and then to flatten out somewhat. The \( 1 + g(CHPAt) \) term adjusts the time-adjusted default rate \( f(t) \) up or down by \( g(CHPAt) \) percent as a function of the change in housing prices relative to a 3 percent baseline.

Ms Steiner’s concerns. Provided the option adjusted spread is rich enough, Ms Steiner is interested in the residual tranche. She does, however, have some structuring concerns.

Ms Steiner is specifically concerned about the impact of the residual tranche on her exposure to real estate risk. She has read a number of reports suggesting the possibility of a bubble in US real estate prices. Ms Steiner has asked Brad to structure the residual tranche (or combine it with a separate option) in a way that will reduce her future cash flow exposure to real estate in down markets. As a possible metric, she has proposed the correlation between (a) the ending accumulated total cash flows from her position in the residual tranche and (b) the corresponding HPA. If this is not the right metric, she is also willing to consider some variation on this metric. Given a choice between a simple real estate hedge and a complicated hedge with wild exotic path dependence – and lots of model risk, she says – Ms Steiner prefers a simple hedge. But she does want something.

More generally, Ms Steiner is also worried about the Value at Risk if she buys the entire residual tranche. The metric she is thinking about for this is the standard deviation of the deal’s ending accumulated payoff (or return). She knows she doesn’t want to take on the residual full risk even after scaling back the real estate risk. Some fraction of the residual’s risk will be enough for her (e.g., say 2/3rds). This worries Brad. He really wants Ms Steiner to buy as much of the residual tranche as possible. He needs to free up Wright Structured Product’s capital for his next deal. If he can’t sell all of the residual, Brad wonders how to modify the residual tranche so as to maximize the amount of the residual cash flows he sells Ms Steiner subject to a given cumulative 10 year cash flow (or return) volatility target.

Brand knows that for some tasks he should simulate cash flows under the RN dynamics while for others he should use the objective dynamics. “Which one is right for the structuring part,” he wonders.

Real estate market conditions. The real estate price process is an input into the future mortgage default and prepayment processes. As a starting point for calibrating his housing prices, Brad has historical data for the FHLBB housing price index in Exhibit 2. See winterland-info.xls for the raw data. Brad also has seen survey data estimating that the average US rental rate is 4 percent of assessed property values.

“OK, real estate is just another asset, but how should I think about ‘dividends,’” puzzles Brad.
Treasury market conditions. Brad intends to use a one-factor Hull & White model for the spot Treasury rate process. Exhibit 3 gives the current term structure for Treasury rates, some historical data, and Wright Research’s parameter calibration. See winterland-info.xls for the raw data.

Intermediate steps. This year’s case lays out some structuring goals, but leaves it up to you how to achieve them. There is lots of room for creativity in your structured product design. To help the judges evaluate the technical accuracy of your valuation independently of your specific structuring solution, we will start the general Q&A by asking you to provide numerical answers to the following intermediate steps:

- What are your valuations for the underlying mortgages themselves and for the plain vanilla residual tranche given an option adjusted spread of 0 (i.e., at Treasury flat)?

- How does the value of the plain vanilla residual tranche change one day after origination given a 100 basis point increase in the 1 year Treasury rate (i.e., keeping the mortgage fixed rate and all of the various floating rate spreads unchanged and also keeping time effectively unchanged)?

Some hints and questions to think about.

- You can simulate annual time steps as a reasonable numerical simplification.

- The default and prepayment models are clearly simplistic. Based on economic and finance first principles, what changes would you want Wright Research to investigate?

- Competition Format. The format is the same as in previous years.

- You should give Ms Steiner (and the judges) a written “term sheet” outlining the key terms of your proposal (e.g., pricing, payoff rule) at the start of your presentation.

- Each presentation will begin with a 15 minute long formal sales pitch addressed to the client. Ms Steiner, while knowledgeable about derivatives, is primarily interested in how your proposal will help solve her business problem. Questions from the judges during the sales pitch will be from the client’s perspective.

- A single spokesperson can represent the team or more than one presenter can be involved as you see fit. In past years, presentations have been in PowerPoint, but you may use other media if you choose.

- Following the sales pitch, the judges will have 25-30 minutes for general Q&A about the details of modeling and technical implementation and about the profitability of the deal and risk management issues at Wright Structured Products. During the Q&A the judges are no longer limited to playing the role of a client.

- The judges may ask questions about alternative parameter values. Bring your spreadsheet (or whatever numerical package you use) to the presentation so you can plug them in your model and discuss them.
Judging criteria. Teams will be evaluated on business intuition, marketing quality, structuring creativity, and technical proficiency. Some specific considerations are:

- **Salesmanship.** The sales pitch should focus on the “forest” (the business drivers behind your proposal) rather than the “trees” (modeling details).

- **Pricing and structure.** Completing the analysis (i.e., structuring and pricing your proposal) is clearly a necessary first step. You should also be able to explain the pros and cons of your modeling choices.

- **Clarity.** When working with technically complicated financial products, clarity and intuition are vital.

Good luck!
### Exhibit 1

**Summary information about HC-7 securitization**

- Characteristics of HEL mortgages in pool

<table>
<thead>
<tr>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial principal</td>
<td>$1.0 billion</td>
</tr>
<tr>
<td>Number of individual mortgages</td>
<td>6,500</td>
</tr>
<tr>
<td>Maturity</td>
<td>10 years</td>
</tr>
<tr>
<td>Principal payment schedule</td>
<td>Balloon at maturity</td>
</tr>
<tr>
<td>Initial prepay freeze-out window</td>
<td>3 years</td>
</tr>
<tr>
<td>Type</td>
<td>Fixed-floating hybrid</td>
</tr>
<tr>
<td>Length of initial fixed rate period</td>
<td>3 years</td>
</tr>
<tr>
<td>Weighted average initial fixed rate</td>
<td>7.31 percent per annum</td>
</tr>
<tr>
<td>Floating index rate</td>
<td>1 year Treasury</td>
</tr>
<tr>
<td>Reset</td>
<td>Annually</td>
</tr>
<tr>
<td>Average floating rate spread</td>
<td>5 percent per annum</td>
</tr>
<tr>
<td>Floor on floating rate</td>
<td>Initial fixed rate at origination</td>
</tr>
<tr>
<td>Servicing fee</td>
<td>50 basis points</td>
</tr>
</tbody>
</table>

- Characteristics of ABS bonds issued in Trust HC-11 securitization

<table>
<thead>
<tr>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial total bond principal</td>
<td>$990 million</td>
</tr>
<tr>
<td>Initial over collateralization</td>
<td>$10 million</td>
</tr>
<tr>
<td>Maturity</td>
<td>10 years</td>
</tr>
<tr>
<td>Type</td>
<td>Floating</td>
</tr>
<tr>
<td>Floating rate index</td>
<td>1 year Treasury</td>
</tr>
<tr>
<td>Reset</td>
<td>Annually</td>
</tr>
<tr>
<td>Average ABS spread over Treasury</td>
<td>1.5 percent per annum</td>
</tr>
<tr>
<td>Available funds cap</td>
<td>Actual mortgage interest received net of fees</td>
</tr>
<tr>
<td>Private mortgage insurance</td>
<td>None</td>
</tr>
</tbody>
</table>
Exhibit 2
Real estate market information

Historical US Home Prices

Date

Index

See spreadsheet winterland-info.xls for raw data
Exhibit 3
Treasury information

- Current US Market Term Structure for Zero-Coupon YTM

<table>
<thead>
<tr>
<th>Maturity (yrs)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero YTM</td>
<td>0.0331</td>
<td>0.0352</td>
<td>0.0395</td>
<td>0.0452</td>
<td>0.0485</td>
<td>0.0497</td>
<td>0.0503</td>
<td>0.0503</td>
<td>0.0501</td>
<td>0.0502</td>
</tr>
</tbody>
</table>

- Historical Treasury rates

See spreadsheet winterland-info.xls for raw data

- Wright Research Parameters for RN Hull & White Process $dr_t = \alpha[\theta(t) - r_t] \, dt + \sigma \, dw_t$

Rate $r_t$ in decimal form: 0.01 = 1 percent

$\alpha = 0.5$
$\sigma = 0.0075$ (i.e., 75 basis points)

- Wright Research Parameters for RN Hull & White Process $dr_t = \alpha[\theta(t) + \text{POR} - r_t] \, dt + \sigma \, dw_t$

POR = 0.0015