**Millennium Aggressive Bond Fund**

“Just one last thing to do and then the weekend is mine,” thought Brad as he picked up his phone to call Noor Al-Hasan at the end of a hard week.

Brad Kaiser works in the Client Strategies Group of Wright Derivatives, a AAA rated subsidiary of WrightBank. Ms Al-Hasan is the lead portfolio manager for the Millennium Aggressive Bond mutual fund at Millennium Asset Management Inc. Exhibit 1 gives descriptive statistics for the Millennium Bond Fund.

During their conversation, Ms Al-Hasan explained that the Millennium Fund has sizeable positions in four corporate bonds, two from the airline industry and two from the insurance industry. Exhibit 2 gives details about these bonds. The recent weakness in the transportation industry in addition to some rating downgrades of insurance bonds has made Ms Al-Hasan nervous about her credit risk exposure. While she is not overly worried that all four bonds will default, she does worry that some of them may default over the next five years.

Before approaching Brad, Ms Al-Hasan spoke with Rob Dudley at Wrong Brothers and Co.. Mr. Dudley advised her to enter into a first-to-default swap in which Wrong Brothers would pay the Millennium Fund par-less-recovery on the first bond in the portfolio to default. The notional principal for the swap would be chosen by Ms Al-Hasan. In return, Millennium would make annual payments of a contractually agreed upon “swap spread” applied to the notional. These payments would terminate in the event of a default. Ms Al-Hasan is dissatisfied with Mr. Dudley’s proposal on two accounts.

First, the Millennium Fund sometimes trades out of bond positions before maturity. In particular, a bond might be sold, not only as part of the active portfolio management process, but also if annual redemptions exceed new inflows into the fund. While the former motive is unpredictable ex ante, Ms Al-Hasan has found that cash inflows and outflows do have a systematic component. In particular, net flows from investors are sensitive to, among other things, changes in interest rates. Whenever the one-year Libor rate rises by 50 basis points (or more) relative to the previous one-year rate a year before – thereby depressing bond prices and the fund’s realized return – the Millennium fund experiences, on average, a 10 percent net reduction of funds under management (i.e., relative to what would have happened otherwise). Thus, in the event that interest rates

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1 Prepared by Duane Seppi and Chris Telmer. The persons and events in this case are fictional and intended solely as a basis for discussion. Any similarities with actual persons or events are coincidental. The case was first used in the 2003 Financial Engineering Case Competition organized by the Tepper School of Business and sponsored by Lehman Brothers and Appaloosa Management, L.P. This case does not necessarily reflect the views of any of the sponsors. We thank David Heike, Stefano Risa and particularly Marco Naldi for helpful conversations. Richard Stanton and Burton Hollifield gave helpful comments on early drafts of this case. © 2003 Duane Seppi and Chris Telmer.

2 Just to be clear, Ms Al-Hasan estimates that if the one-year Libor rate $R_t$ at the start of year $t$ is 50 b.p. greater than the previous one-year rate $R_{t-1}$ at the start of year $t-1$, then the amount $M_t$ invested in the Millennium Fund at the start of year $t$ is 10 percent less than it otherwise would be. In this discussion, the
rise in the future, the four bond positions in question will likely be reduced on a pro rata basis. Ms Al-Hasan is concerned about paying for “excessive” protection against defaults on those bonds that she might well have already sold.

On the other hand, although falling future interest rates lead to predictable net inflows, Ms Al-Hasan has the entire bond market to choose from when investing any incremental funds in this scenario. She says it is unlikely she would choose to increase her positions in these four particular bonds.

A second complication, in Ms Al-Hasan’s opinion, is that the size of her positions differs significantly across the four bonds. Under the standard first-to-default structure proposed by Wrong Brothers, she must choose the same notional principal for all four of the bonds in the swap. In other words, the notional principal per bond is identical irrespective of the size of her positions in the bonds. She worries that the standard structure does not suit her needs. Does this mean she gets the same payoff if a big bond position defaults (before taking differences in recovery rates into account) as if a small bond defaults? And what if one of the smaller positions defaults first followed later by a default on a larger position? What then? Ms Al-Hasan is frustrated with Mr. Dudley’s lack flexibility on these points. He has said that she will just have to choose an intermediate notional size and “split the difference.” Ms Al-Hasan wonders whether a more tailored product might better suit her while still keeping some of the cost advantages of a generic first-to-default structure.

At the end of their conversation, Ms Al-Hasan asks Brad to do some “out of the box” thinking. She’d like his opinion on both the Wrong Brothers proposal as well as his thoughts on more exotic structures that address her two qualms.

**Corporate bond default dynamics.** To model default dynamics, Brad plans to simulate “stopping times” for bond defaults using the Normal copula technique. He realizes that, in order to do this, he must obtain marginal default probabilities for each of the issuers of the bonds in the Millennium portfolio. He also realizes that he will need information on the asset correlations of the bonds.

Regrettably, multiple-maturity information on these specific issuers is not available. The only information Brad has for the default probabilities is the market term structure for BBB and BB bonds (i.e., averaged across different issuers of the same rating) together with the only liquid point on the default swap curves for these specific issuers: the 5-year point. Exhibit 3 provides explicit details.

As to the correlations, Rob is certain that defaults on the two airline-industry bonds are highly correlated, while defaults on the two insurance bonds are somewhat less correlated. Wright Research estimates correlations of 0.80 and 0.50 for the airline and insurance bonds, respectively, and, due to general macroeconomic influences, 0.15 one-year Libor rate is simply a convenient proxy for bond market conditions more generally. If you simulate annual interest rates, you can assume that fund withdrawals occur at the end of the year.
between the two industries. Of course there is uncertainty about these point estimates
given standard errors for each of the correlations of 0.05.

**Treasury market conditions.** Brad intends to model interest rates using a variation of a
one-factor Hull and White style model\(^3\) in which the spot rate dynamics are:

\[
dr_t = \alpha (\theta(t) - r_t) \, dt + \nu(t) \, dw_t.
\]

Exhibit 4 gives the current market Treasury term structure and implied
local relative volatilities \(\nu(t)\) calibrated from cap and swaption prices. Wright Research
recommends using a speed of mean-reversion \(\alpha\) equal to 0.5. To get started, Brad
assumes there is no significant correlation between interest rates and defaults.

**Some hints**

- The case leaves room for creative structuring. Look for a structure that addresses
  your client’s needs in a way that is intuitive and which you will be able to explain to
  the client.

- The most important thing in your numerical computations is to have a *conceptually*
  sound approach. Given the short time available, it is OK to make reasonable
  simplifications. For example, it is OK to simulated one-year Libor rates using annual
  time steps. That said, more realistic modeling is preferred to less.

- The case is do-able in Excel, but you are free to use any other generic software (e.g.,
  Matlab) or Monte Carlo add-ins (e.g., @RISK or Crystal Ball).

- While hedging will not be the sole focus of the general Q&A, you may get questions
  from your boss at Wright Derivatives about your hedging strategy if you get this deal.

**Presentation Format.** Your team will have 15-20 minutes for a formal sales pitch to the
client. Ms. Al-Hasan, while knowledgeable about derivatives, is primarily interested in
how your proposal will help solve her business problem. Any questions during the sales
pitch will be from the client’s perspective. After the sales pitch portion, you can briefly
review any non-client valuation or business issues you feel are important. The rest of the
time will be general Q&A about the profitability of the deal (for Wright Derivatives), risk
management, and details of modeling and technical implementation. At least two team
members must be involved in the sales pitch and technical overview presentation. You
can also have more than two presenters if you so choose.

- You should provide Ms Colt with a *written* “term sheet” outlining the key terms of
  your proposal (e.g., pricing, payoff rule) at the start of your presentation.

\(^3\) It is fine to use a different interest rate model if that is easier. To help with your calibration and also for
debugging purposes, the terminal one-year spot rate in five years in our Monte Carlo had a minimum of 1.5
percent, a maximum of 5 percent and a standard deviation of one half percent.
• You may be asked questions about alternative parameter values. Bring your spreadsheet (or whatever numerical package you use) to the presentation so you can plug them in your model and discuss them.

**Evaluation criteria.** Teams will be evaluated on business intuition, marketing quality, structuring creativity, and technical proficiency. Some specific considerations are:

• *Salesmanship.* The sales pitch in the first 20 minutes should focus on the “forest” (the business drivers behind your proposal) rather than the “trees” (modeling details).

• *Pricing and structure.* Completing the analysis (i.e., structuring and pricing your proposal) is clearly a necessary first step. You should also be able to explain the pros and cons of your modeling choices.

• *Clarity.* When working with technically complicated financial products, clarity and intuition are highly valued.

Good luck!
Exhibit 1
Key facts about the Millennium Corporate Bond Fund

Founded 1983

Funds under management $2.7 billion

Annual SE of funds under management $350 million

Annual alpha 75 basis points
(benchmark = LB US Aggregate Bond Index)

Annual portfolio turnover ratio 28 percent

Exhibit 2
Description of Individual Bonds

<table>
<thead>
<tr>
<th>Industry</th>
<th>Laker Airways</th>
<th>Fife Pacific</th>
<th>Fennerio Group</th>
<th>Willoughby Inc.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Airline</td>
<td>Airline</td>
<td>Insurance</td>
<td>Insurance</td>
</tr>
<tr>
<td>Rating</td>
<td>BB</td>
<td>BB</td>
<td>BBB</td>
<td>BBB</td>
</tr>
<tr>
<td>Years-to-maturity</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Annual coupon rate*</td>
<td>8.50</td>
<td>8.00</td>
<td>7.75</td>
<td>7.125</td>
</tr>
<tr>
<td>Recovery rate (percent)**</td>
<td>30</td>
<td>30</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>5yr Default Swap Spread (b.p.)</td>
<td>754</td>
<td>520</td>
<td>210</td>
<td>180</td>
</tr>
</tbody>
</table>

| Millennium positions (USD millions in bond principal) | 50 | 100 | 100 | 75 |

* Annual percentage coupon rate. Coupons are paid semiannually.

** Recovery rates are expressed as a percentage of the principal plus accrued interest at the time of default. For example, if the principal is 100, accrued interest is 8/2 = 4% and the bond pays off 30, then the recovery rate R satisfies R = 30/104.
### Exhibit 3
Libor Par-Floater Credit Spreads (basis points)

<table>
<thead>
<tr>
<th>Years</th>
<th>US All BBB Libor Spread</th>
<th>US All BB Libor Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>133.48</td>
<td>431.92</td>
</tr>
<tr>
<td>2</td>
<td>146.67</td>
<td>431.37</td>
</tr>
<tr>
<td>3</td>
<td>150.56</td>
<td>432.24</td>
</tr>
<tr>
<td>4</td>
<td>162.65</td>
<td>431.07</td>
</tr>
<tr>
<td>5</td>
<td>174.74</td>
<td>429.89</td>
</tr>
</tbody>
</table>

### Exhibit 4
Spot Interest Rates from Libor Swap Rate Curve (percent)

<table>
<thead>
<tr>
<th>Maturity (yrs)</th>
<th>Zero rates</th>
<th>Local Annualized Volatilities *</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2800</td>
<td>n.a.</td>
</tr>
<tr>
<td>2</td>
<td>1.5122</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>1.7748</td>
<td>0.35</td>
</tr>
<tr>
<td>4</td>
<td>2.0232</td>
<td>0.40</td>
</tr>
<tr>
<td>5</td>
<td>2.2503</td>
<td>0.40</td>
</tr>
</tbody>
</table>

* 0.35 = an absolute annualized local interest rate SD of 35 b.p.