Aigner Mortgage Services[1]

"Sharon Martinez called while you were out." Brad Kaiser put down his lunch and picked up his telephone.

Brad Kaiser works in the Client Financial Strategies Group at Wright Derivatives Co., a AAA rated subsidiary of WrightBank Inc. specializing in swaps and other derivative securities transactions. Sharon Martinez is the CFO at Aigner Mortgage Services (AMS) Inc.

AMS is a medium-size player in the mortgage servicing business. When mortgages are packaged together to create pass-throughs and other mortgage backed securities, a portion of the monthly mortgage payment is retained by the mortgage servicer as compensation for collecting and distributing the principal and interest payments from the underlying pool of mortgages.

AMS's monthly servicing fee used to be an annualized 50 basis points per dollar of principal, but two years ago it fell, due to greater competition, to 40 basis points. Thus, the monthly servicing revenue on $1 million dollars of old principal (i.e., on mortgages securitized more than two years ago) is $1,000,000 × .005/12 or roughly $417. On the other hand, the corresponding revenue per million dollars of new mortgages is only $1,000,000 × .004/12 or $333 per month.

Since AMS's servicing income is based on the monthly outstanding principal balances of the mortgages it services, prepayments in excess of contractually scheduled amortization are a major issue. Ms Martinez estimates that mortgages will prepay at an annualized constant prepayment rate (CPR) of 5 percent at current or higher interest rate levels. However, she expects prepayments to accelerate to a total of 10 percent CPR whenever the one-year zero-coupon Treasury rate – which she uses as a reference rate – is 50 b.p. below its current level (i.e., as home owners refinance their mortgages to take advantage of lower rates). Similarly, a one-year Treasury rate that is 100 b.p. lower than today leads to a 15 percent CPR and a 200 b.p. lower Treasury rate leads to a 25 percent CPR (which she believes is the maximum possible CPR). In between these rates she uses linear interpolation to calculate the CPR.

In essence, AMS's revenue from servicing mortgages currently in its pool is like an IO (interest only) strip. When interest rates are high, prepayments slow down (i.e., to a projected minimum of 5 percent CPR) and more of the principal balance on AMS's current mortgage pool is carried forward into the future. This, in turn, keeps future fee income high. However, if interest rates are low, prepayments accelerate (i.e., to a maximum of 25 percent CPR), thereby reducing the principal balance going forward on which future fees are calculated.

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Of course, if prepayments are associated with refinancings, then AMS will eventually pick up its share of the new mortgages, thereby restoring its principal. However, while Ms Martinez is confident that AMS would eventually recapture a comparable share of subsequent refinancings, she also knows that the fee AMS is likely to receive on new refinancings is lower than the fee it currently receives on most of its existing pool. Thus, there are interest rate related path dependencies – once again, similar to those in an IO strip – in AMS's total revenue. In particular, its revenue depends on the split between old and new mortgages in its servicing pool as well as on the total principal balance.

AMS's cost structure is such that almost all of its costs are fixed costs (e.g., computers, staff salaries, office space). Consequently, the marginal cost of servicing an additional million dollars of mortgage principal is almost zero. Dividing AMS's total fixed costs by the principal of mortgages it currently services gives annual allocated fixed costs of roughly 30 basis points per dollar of principal given AMS's current pool size.

Sharon Martinez explained to Brad Kaiser that she is concerned about the possible effect of a drop in interest rates on AMS's net servicing profit given the fee differential on new versus her current mortgages. Although the Federal Reserve raised short-term rates last week, Ms Martinez is worried that the economy may be weaker than it appears. If the economy and interest rates were to weaken suddenly, her mortgages would prepay faster.

Competitor's proposal. Before calling Wright Derivatives, Ms. Martinez previously spoke with Rob Dudley at Wrong Inc. Mr. Dudley advised AMS to buy a customized interest rate floor with a deterministic but time-varying notional. In each month t, AMS's payoff on the floorlet for that month would be:

$$P(t) \times (.10/12) \times (.001/12) \times \max\{0, r(0) - r(t)\} \times 100$$

where the strike rate r(0) is the initial one-year zero-coupon rate at the inception of the floor and r(t) is the future (random) one-year zero rate at the start of month t (i.e., both are in decimal form so 7 percent = .07) and P(t) is the projected remaining principal balance on old mortgages in month t given the contractual amortization plus a constant 5 percent CPR but *excluding* any accelerated prepayment if rates drop between now and date t. In words, the monthly payoff under Mr. Dudley's proposal is the product of:

- The projected monthly principal with a constant 5 percent CPR times
- The monthly acceleration in prepayments per percent the floor is in-the-money times
- The monthly old vs. new servicing differential.
- A put on the month-t one-year Treasury rate struck at-the-money today times
- 100 to convert decimal rates into percentage rates.

Thus, if interest rates are low in month t, then that month's floor will expire in-the-money, thereby offsetting, to a certain extent, AMS's forgone fee income in that month.

Ms. Martinez is dissatisfied with Mr. Dudley's proposal on two counts. First, its payoffs do not reflect the *dynamic* path-dependent interest rate dependence in AMS's outstanding principal balances and, hence, in its servicing income. Second, she is leaning towards a swap type transaction (unless there is a good reason not to go that

way) to avoid any current cash flows. Given her dissatisfaction, Ms. Martinez asks if Brad Kaiser to come up with a strategy which is better suited to AMS´s needs.

Current market conditions. After working out the general structure of his proposal for AMS, Brad consults with his colleagues in the WD Fixed Income Research group (a) to make sure that his valuation is consistent with arbitrage-free pricing and (b) to work out a hedging strategy since Wright Derivatives will take the other side of the trade.

Current interest rate conditions are given in Exhibits 1 and 2. Exhibit 1 gives three-month Treasury forward rates. For example, 6.236 is the three-month forward rate six months forward (i.e., between 0.5 and 0.75 years) and 6.465 is the three-month forward rate nine months forward (i.e., starting in 0.75 years). The current six-month annualized Treasury spot rate is 5.548. For simplicity, just add a constant spread of 25 b.p. to the Treasury zero yields to get the corresponding (zero-coupon) swap rate curve.

Exhibit 2 gives annualized market interest rate cap forward volatilities. For simplicity, just use the mid points of the ranges. The 1×2 midpoint cap vol of 16.5 implies a volatility for the future *log* one-year rate in one year (i.e., the log of the rate that will be set in 12 months to discount cash 24 months from now) of 16.5 percent (i.e., $\sigma_2 = .165$). Similarly, the 2×3 vol of (16.8+17.8)/2 = 17.3 implies a volatility for the future log one-year rate in two years (i.e., which will be set in 24 months) of 17.3 percent (i.e., $\sigma_3 = .173$), etc. The 3×4 vol is interpreted analogously for the spot rate set in three years. In a Black-Derman-Toy model the (relative) log rate volatilities are the appropriate volatility input.

In the Hull and White or Ho and Lee models, however, what matters are the *absolute* volatilities of (non-log) interest rate levels.[2] One simple way to estimate this is to multiply the (relative) cap vols by the corresponding current forward rates for each year. For example, if the forward rate is 7 percent and the lognormal (relative) future spot vol is 18 percent, then the corresponding implied (absolute) future interest rate vol σ would be .07 × .18 = 0126 or 126 b.p.

In principle, both the parameters σ (in HW and HL) and *a* (in HW) can be chosen by comparing the future spot rate volatilities they induce with the derived market (absolute) volatility structure.[3] However, WD Research has found that in Hull and White implementations just setting the parameter *a* to be 0.1 (as in Hull Ch. 17) usually seems to work fine.

Hints. For purposes of the case, you can make two simplifying assumptions. First, assume that prepayments and servicing fees are received *annually* rather than on a monthly basis. In particular, homeowners look at the interest rate at the *beginning* of each year and make any prepayments depending on whether or not they decide to refinance their mortgage (i.e., thereby determining that year's CPR). Thus, mortgages

[2] What is going on here is that the volatility function $\sigma(t)$ in the BDT and in the lognormal cap model refers to the variability of interest rates as a relative proportion of their level while in the HW and HL models volatility σ is measured in absolute terms.

[3] See Hull, Chapter 17 for more on the relation between a and σ and future interest rate volatility term structures.

with a million dollars of outstanding principal and a 15 percent CPR experience prepayments of $1,000,000 × .15 or $150,000. AMS's servicing revenue (i.e., based on the post-prepayment beginning-of-the-year composition of its portfolio) is then received at the *end* of the year. In addition, this assumption means that your interest rate trees and/or Monte Carlo simulations can also be set up with one-year (rather than monthly) time intervals.

As a second simplifying assumption, focus your analysis on a single *subpool* fixed rate mortgages which currently have ten years until maturity and a remaining current principal balance of $100 billion.

In preparing for your presentation, you will want to:

- Model the portion of AMS's current unhedged servicing cash flows on the ten-year subpool which are at risk given the fee differential on current versus old mortgages and their dependence on interest rates. How serious are these risks relative to AMS's servicing revenue? Relative to its servicing net profit?

- Work out a suitable structure to protect AMS against this risk. What advantages, if any, might a swap have over options on an IO? If you design a swap, what would you recommend AMS swap into?

- Be able to explain how your solution is better than Rob Dudley and Wrong Inc's proposed structure.

- Price your proposal using a single-factor interest rate model. If, for some reason, not enough information is provided in the case to completely parameterize the interest rate model you choose, just assume reasonable values for the variables you cannot pin down after using the data provided and proceed.

- Understand and be able to explain the risk exposures to Wright Derivatives if they take the other side of the trade on the AMS deal. For example, how would the value of an unhedged position by Wright Derivatives change in response to various changes in the zero Treasury curve? If the volatilities of interest rates changed?

- In terms of credit risk, assume that AMS has an AA rating. Without doing any specific calculations, how should Wright Derivatives think about AMS counterparty risk in this context? In particular, what is the *worst* pattern of cash flows Wright Derivatives could receive (as the counterparty) if AMS were to default at some point?

- Describe the hedging strategy you would recommend Wright Derivatives follow to hedge its various risks.

- Recommend an initial hedge position in Treasuries zeros for Wright Derivatives to take today to hedge its term structure risk from this deal.

Presentation. Your team will have 20 minutes for a sales pitch to Ms. Martinez. You may assume that your client is knowledgeable about interest rate modeling, but that she is ultimately most interested in how your analysis will help her to solve her business

problem. A single spokesman can be delegated to represent the entire team or more than one presenter can be involved as you see fit. Your sales pitch will be evaluated in terms of the following criteria

- *Appropriateness of the structured product.* You have a fair amount of latitude in designing the structured product for AMS. Why did you choose the structure you did? Intuitively, why is the Wrong structure poorly conceived? As part of your presentation you should distribute a *written* "term sheet" outlining the key features of your proposal (e.g., price, payoff rule).

- *Salesmanship.* While you want to understand the details, it is important to be able to rise above them in your presentation. Focus on the forest rather than the trees. You are selling an idea either to Ms Martinez. The technical specifics of your analysis are a *means* to that end, rather than an end in and of themselves. Sell the deal. Sell your team. Sell your firm.

- *Clarity of the presentation and explanations.* When dealing with technically complicated products, clarity and intuition are particularly important.

After your formal sales pitch you will have an opportunity to explain the technical details of your valuation. You should also discuss them, along with any relevant business considerations for Wright Derivatives, in your memo to your boss.

Good luck!

Exhibit 1(a): Treasury Spline Implied Forward Short Rates

Years	Rate	Years	Rate
0.5	6.236	7	7.056
0.75	6.465	8	7.0565
1	6.558	9	7.0565
1.25	6.694	10	7.057
1.5	6.793		
1.75	6.864		
2	6.915		
2.25	6.951		
2.5	6.976		
2.75	6.995		
3	7.009		
3.25	7.02		
3.5	7.032		
3.75	7.043		
4	7.055		
5	7.055		
6	7.056		

Exhibit 2: Cap Forward Volatilities

1x2	16.00-17.00
2x3	16.80-17.80
3x4	17.00-18.00